I Semestral Exam 2002-2003
B. Math. Hons. III year
Complex Analysis

Date: 20-11-2002 **Max. Marks: 60** **Instructor: A. Sitaram**

Answer all questions. Each question carries 10 marks.

You may use your classroom notes in the exam.

1. Decide if $f(x) = \frac{x+y+z}{(x^2+y^2+z^2)^{3/2}}$ is harmonic in $R^3 \setminus \{(0,0,0)\}$. Justify your answer.

2. f is harmonic on R^3 and $\left|\frac{1}{f(x,y,z)}\right| \geq x^2 + y^2 + z^2$ if $x^2 + y^2 + z^2 \geq 3$. Prove that $f \equiv 0$.

3. Construct an explicit map of $\mathbf{C}$ into $\mathbf{C}$ of the form $z \longrightarrow \frac{az+b}{cz+d}$, which maps $\{z : z \text{ above the line } y = x\}$ onto $\{z : |z-5| < 3\}$.

4. $f(z)$ is holomorphic on $\mathbf{C} \setminus \{0\}$ and has a simple pole at 0. If $f(z) \to 0$ as $|z| \to \infty$, prove that $f(z) = \frac{constant}{z}$.

5. Evaluate $\int\limits_{|z-8|=1} \frac{dz}{(2Log\ z-3)(z-8)}$.

6. Let $f(z) = \sum\limits_{n=1}^{\infty} \frac{1}{z^2-n^2}$ and $S_N(z) = \sum\limits_{n=1}^{N} \frac{1}{(z^2-n^2)}$. Prove that $S_N \to f$ uniformly on compact subsets of $\mathbf{C} \setminus \mathbb{Z}$. Is f holomorphic? Justify your answer.

7. Let $f(z) = Log(e^{i\pi/4} z)$. Find the maximal open set Ω on which f is holomorphic. Find the value of $f(x+ix), x > 0$.