Physics I
ISI B.Math
Midterm Exam : September 22, 2022

Total Marks: 70
Time : 3 hours

Answer all questions:

1.(**Marks = 7 + 7 = 14):**

A particle of mass m moves in one dimension under a conservative force with potential energy

$$V(x) = \frac{cx}{x^2 + a^2}$$

where $a, c \geq 0$.

(i) Find the position of stable equilibrium and the period of small oscillations about it.

(ii) If the particle starts from this point with velocity v, find the range of values of v for which it (a) oscillates (b) escapes to $-\infty$ (c) escapes to $+\infty$.

2.(**Marks = 10 + 4 = 14):**

(i)Which of the following forces are conservative ? If conservative, find the potential energy $U(\mathbf{r})$. a, b, c are constants
(a)$F_x = y, F_y = -x, F_z = 0$
(b) $F_x = ayz + bx + c, F_y = axz + bz, F_z = axy + by$
(c) $F_x = -ze^{-x}, F_y = \ln z, F_z = e^{-x} + \frac{y}{z}$

(ii) Is the work done by the force $\mathbf{F} = \frac{a \sin t}{r^2}\hat{\mathbf{r}}$ on a particle of mass m in moving it between two fixed coordinate locations independent of path ? Is the total mechanical energy conserved in the process ? Explain.

3. (**Marks = 7 + 7 = 14**)

a) A particle with polar coordinates r, θ which are functions of time t is moving in a plane. The velocity and acceleration of the particle can be written in plane polar coordinates as $\mathbf{v} = v_r\hat{\mathbf{r}} + v_\theta\hat{\theta}$ and $\mathbf{a} = a_r\hat{\mathbf{r}} + a_\theta\hat{\theta}$. Find $v_r, v_\theta, a_r, a_\theta$ as functions of $(r, \theta, \dot{r}, \dot{\theta}, \ddot{r}, \ddot{\theta})$

b) An insect flies on a spiral trajectory such that its polar coordinates at time t are given by $r = be^{\Omega t}, \theta = \Omega t$. where b and Ω are positive constants. Find the velocity and acceleration vectors of the insect at time t and show that the angle between them is $\frac{\pi}{4}$

4. (**Marks = 12 + 2 = 14**)

A body is projected vertically upwards with speed u and moves under uniform gravity in a medium that exerts a resistance force proportional to the fourth power of its speed and in which the body's

terminal speed is V.

(a) Find the maximum height above the starting point attained by the body.

(b) Deduce that , however large u may be, this maximum height is always less than $\frac{\pi V^2}{4g}$

5. (**Marks** $= \mathbf{6} + \mathbf{5} + \mathbf{3} = \mathbf{14}$)

An electrical circuit consists of an inductance L, resistance R and a capacitance C connected in series with a battery of emf $\mathcal{E}$. The charge passing through the circuit at a time t is given by $q(t)$ and the current $I(t) = \frac{dq}{dt}$. The parameters are such that $R = 2\sqrt{\frac{L}{C}}$. $q = q_0$ and $I = 0$ at $t = 0$. Kirchoff's equation around the circuit is given by

$$L\frac{dI}{dt} + RI + \frac{q}{C} = \mathcal{E}$$

a) Solve this equation to find $q(t)$.

b) Now remove the resistance from the circuit and find $q(t)$ with the same initial conditions.

c) Considering the configuration of part b), make a rough sketch of q vs I.