MID-SEMESTER EXAMINATION
B. MATH II YEAR, II SEMESTER 2012-2013
ANALYSIS IV

Max. Score:100 Time limit: 3hrs.

1. Consider the set of polynomials p satisfying the condition $\int_0^1 p(x)dx = 1$ as a subset of $C[0,1]$ (with the usual supremum norm). Is this set totally bounded? Justify. [15]

2. Is the set of all functions of the type $\sum_{j=0}^{N} a_j[\sin(x)]^{2j}$ (where $N \geq 1$ and $a_j's \in \mathbb{R}$) dense in $C([-2,2])$ (with the usual supremum norm)? Justify. [15]

3. Consider the initial value problem $y' = f(x,y), y(0) = 1/3$ where f is continuous function : $[-1,1] \times [-1,1] \to [-3,3]$ which has continuous partial derivative w.r.t. y at every point satisfying $\left|\frac{\partial f}{\partial y}\right| \leq 1$ at every point. Show that this problem has a unique solution on $[-\delta, \delta]$ where $\delta = \frac{2}{9}$. [15]

4. If f is coninuously differentiable on (a,b) and if f' is non-decreasing show that f is convex. [15]

5. Prove that the vector space spanned by $\{z^n : n = 0,1,2,...\}$ is not dense in the space $C(T)$ (where $T = \{z \in \mathbb{C} : |z| = 1\}$ and $C(T)$ is given the supremum metric).

Hints: prove that $\int_0^{2\pi} f(e^{it})e^{it}dt = 0$ for every polynomial $f(z) = \sum_{j=0}^{k} a_j z^j$. Show that this is false for the function $f(z) = \bar{z}$. [20]

6. Show that there does not exist independent elements $f_1, f_2, ...$ in $C[0,1]$ which span $C[0,1]$. [20]

Hint: consider the subspaces spanned by $\{f_1, f_2, ..., f_n\}$ $(n = 1, 2...)$ and apply Baire Category Theorem.