MID-SEMSTER EXAMINATION
II SEMESTER, 2010-2011

B. MATH II YEAR
ANALYSIS IV

Time Limit: 3hrs Max Marks: 80

1. Let $X = C[0,1]$, the space of all continuous functions from $[0,1]$ into $\mathbb{R}$ with the metric $d(f,g) = \sup_{0 \le x \le 1} |f(x) - g(x)|$. Consider the following three subsets of X :

1) $K_1 = \{1, x, x^2, x^3, ...\}$

2) $K_2 =$ set of all polynomials with rational coefficients

3) $K_3 = \{p(\sin(7x) : p \text{ is a polynomial}\}$

Which of these sets are compact? Which of them are connected and which are dense in X? Justify your answers. [30]

2. Let X and Y be random variables with values in $[0,1]$ such that $EX^n = EY^n$ for all non-negative integers n. Prove that the distribution functions of X and Y are identical.

Hint: First prove that $Ef(X) = Ef(Y)$ for any $f \in C[0,1]$ and then prove that the same equation holds for $f = I_{[0,a]}$ for any $a \in [0,1]$. [15]

3. Prove that the equation $\frac{dy}{dx} = e^x \sin(y)$ has a unique solution satisfying $y(0) = 1$. [10]

4. Prove that $\int_0^\pi f(x)\sin(nx)dx \to 0$ as $n \to \infty$ for any $f \in C[0,\pi]$. [Do not use Riemann Lebesgue Lemma]. [15]

5. Prove that $\{\sin(nx)\}_{n \ge 1}$ is not equi-continuous in $C[0,\pi]$.

Hint: if it is, prove (using problem 4) above) to show that $\sin(n_k x) \to 0$ uniformly for some $\{n_k\}$; compute $\int_0^\pi \sin^2(n_k x)dx$ to get a contradiction. [10]