B. Math IInd year Analysis Mid Semester 05-03-2007 . Answer all the questions (8 x 5 = 40).

If you are using a theorem/result proved in the class state it correctly. All answers need justification.

1) Let $A \subset C^n$. Show that A is separable.

2) Consider the spaces c_0 and ℓ^∞ with the usual metric. Let $C \subset \ell^\infty$ be a compact set. Show that $c_0 + C$ is a closed set.

3) Let (X, d) me a metric space such that the intersection of any sequence of open sets is open. Show that the topology is the discrete topology.

4) Let (X, d) be a metric space. Show that $A \subset X$ is no where dense if and only if every open set has a open subset disjoint from A.

5) Let $C_b(R)$ denote the space of bounded continuous functions on R with the usual metric. Let $\{f_n\}_{n\geq 1} \subset C_b(R)$ be a bounded and equi-continuous sequence. Suppose for every rational number $lim\ f_n(r)$ exists. Show that $\{f_n\}_{n\geq 1}$ is a Cauchy sequence.

6) Let $E \subset R^n$ be an open set. Let $f : E \to R^n$ be a $\mathcal{C}^1$ function such that $f'(x)$ is non-singular for all $x \in E$. Give the complete details to show that f is an open mapping.

7) Consider the compact Hausdorff space $X = [0,1]^{[0,1]}$. Given $f \in C(X)$ show that there is a countable set $A \subset [0,1]$ such that $x, y \in X$ and $x = y$ on A implies $f(x) = f(y)$. Note that for any $\alpha \in [0,1]$ the evaluation map $e_\alpha \in C(X)$ has this property for $A = \{\alpha\}$.

8) Let X be a compact metric space. Let $I \subset C(X)$ be a proper ideal. Show that there exist a $x_0 \in X$ such that $f(x_0) = 0$ for all $f \in I$.