Indian Statistical Institute, Bangalore

B. Math. IIIrd Year Second Semester

Analysis IV

Back-paper Examination | Date : 11-06-2019

Maximum marks: 100 | Time: 3 hours

(1) Let (X, d) be a complete and totally bounded metric space. Show that (X, d) is compact. [15]

(2) Let $Y = C_{\mathbb{R}}[0, 1]$ be the space of all real valued continuous functions on $[0, 1]$ with supremum norm. (i) Give an example of a subset of Y which is closed and equicontinuous but not bounded. (ii) Give an example of a subset of Y which is bounded and equicontinuous but not closed. (iii) Give an example of a subset of Y which is closed and bounded but not equicontinuous. Prove your claims. [15]

(3) Let V, W be normed linear spaces and let $T : V \to W$ be a linear map. Show that T is continuous iff it is bounded as a linear map. [15]

(4) Compute the radius of convergence around 0 for following power series:
   (a) $\sum_{n=1}^{\infty} \frac{4}{n^2} z^n$;
   (b) $\sum_{n=1}^{\infty} \frac{4}{n^2} z^{2n}$;
   (c) $\sum_{n=1}^{\infty} \frac{4}{n!} z^n$.

[15]

(5) Suppose p is a function with power series expansion $p(z) = \sum_{n=0}^{\infty} p_n z^n$ with radius of convergence $R > 0$ and $p_0 \neq 0$. Show that $q(z) = \frac{1}{p(z)}$ also has a power series expansion around 0. [15]

(6) Compute the Fourier coefficients of the 2π-periodic function h defined by

$$h(t) = 2t, \ -\pi < t < \pi.$$

Use this to obtain an infinite series for the value of π. [15]

(7) Let $C[0, 1]$ be the space of complex valued continuous functions on $[0, 1]$. For f, g in $C[0, 1]$, define $f \star g$ by

$$(f \star g)(x) = \int_0^1 f(t)g(x - t)dt, \ 0 \leq x \leq 1.$$

Show that $\star$ is a commutative and associative binary operation on $C[0, 1]$. Does $\star$ have a unit (identity element)? Prove your claim. [15]