BACKPAPER EXAMINATION
B. MATH II YEAR
ANALYSIS IV
II SEMESTER, 2010-2011

The 7 questions carry a total of 110 marks. Answer as many questions as you can. The maximum you can score is 100. Time limit is 3 hours.

1. Let $f : [0,1] \to [0,1]$ be continuous and bijective. Let $g : [0,1] \to [0,1]$ be a continuous function with $\int_0^1 g(x)[f(x)]^{6n} dx = 0$ for $n = 0, 1, 2, ...$. Prove that $g(x) = 0 \forall x$. [20]

2. Let $f(x,y) = \sin[g(x)y^2], -1 \leq x \leq 1, -1 \leq y \leq 1$ where g is a continuous function with values in $[-1,1]$. Using Picard's Theorem describe a procedure for solving the differential equation $y' = f(x,y)$ with the initial condition $y(0) = 1/2$. [20]

3. Which of the following sequences in $C[0,1]$ are equicontinuous? Justify.

a) $f_n(x) = (1+x^2)^n$

0.1 b) $f_n(x) = (\frac{1}{1+x^2} + 5)^n$

c) $f_n(x) = e^{-ne^x}$ [15]

4. Write down the Fourier series of the function $f(x) = x^2 (-\pi \leq x \leq \pi)$. At what points does it converge? [15]

5. Does there exists a twice continuously differentiable periodic function f such that $f''(x) + f(x) = \sin x$ for all $x \in [-\pi, \pi]$?. Justify. [15]

Hint: use Fourier coefficients.

6. Let $f \in L^2[-\pi, \pi]$ and $f\hat{}(n) = \frac{1}{\sqrt{n}}$. Prove that f is not of bounded variation. [10]

7. Prove that the Fourier series of e^{x^x} $(-\pi \leq x \leq \pi)$ converges at each point of $(-\pi/10, \pi/10)$. [15]