Indian Statistical Institute
II Sem, II Semestral Examination 2008-2009
B.Math.(Hons). II year
Analysis IV (Back Paper)

Date:13-07-2009 Duration: 3 Hours Instructor: B.Bagchi

Max Marks 100

1. Let $\mathcal{H}$ be a Hilbert space.

   (a) If C is a non-empty closed and convex subset of $\mathcal{H}$ then show that C contains one and only one element of smallest norm.

   (b) Use part (a) to show that for any closed linear subspace M of $\mathcal{H}$, we have the decomposition $\mathcal{H} = M \bigoplus M^{\perp}$. [8 + 12 = 20]

2. (a) If K is a compact metric space and $T : K \longrightarrow K$ is an isometry then show that T is onto.

   (b) If $\mathbb{V}$ is a finite dimensional normed linear space and $T : \mathbb{V} \longrightarrow \mathbb{V}$ is an isometry then use part (a) to prove that T is onto. [5 + 15 = 20]

3. Let B be a real Banach space and $T : B \longrightarrow B$ be an isometry such that $T(0) = 0$. Suppose T is onto. Then show that T is linear. [20]

4. Show that any metric space embed isometrically as a dense subspace of a complete metric space. [20]

5. Let X be a locally compact metric space and let V_n $n = 1, 2, 3, ...$ be a sequence of dense open subsets of X. The show that $\bigcap_{n=1}^{\infty} V_n$ is dense in X. [20]