

TOPOLOGY II - MID-SEMESTRAL EXAM

Time : 2 hours

Max. Marks : 40

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

- (1) Decide whether the following statements are *True* or *False*. Answers without correct and complete justifications will not be given any marks.
 - (a) S^n is a deformation retract of $\mathbb{R}^{n+1} - 0$.
 - (b) The universal cover of a connected graph is contractible.
 - (c) A finite index subgroup of a free group of rank 2 is finitely generated.
 - (d) If X and Y are connected spaces having homeomorphic universal covers, then X and Y are homeomorphic.
 - (e) Every map $f : \mathbb{R}P^2 \rightarrow X$ into a connected graph is null homotopic. [4 × 5 = 20]
- (2) Construct a connected covering space of $S^1 \vee S^1$ corresponding to the normal subgroup of $\pi_1(S^1 \vee S^1)$ generated by a^2, b^2 . Here a, b have the usual meaning. [10]
- (3) Describe the connected coverings of $\mathbb{R}P^2 \times \mathbb{R}P^2$ up to covering space isomorphism. [10]