Topology
M. Math. I
Mid-Term Examination

Instructions: All questions carry equal marks.

1. Let $X$ and $Y$ be spaces and $A \subset X$ be a subspace. Let $f : A \rightarrow Y$ be a continuous map. If $Y$ is Hausdorff, then show that there exists at most one extension of $f$ to the closure $\overline{A}$ of $A$. Give an example to show that this statement is false if $Y$ is not Hausdorff. Justify your answer.

2. Let $X_n = [0, \frac{1}{n}]$. If $\prod X_n$ is given the product topology, then prove that every continuous real valued function $f : \prod X_n \rightarrow \mathbb{R}$ is bounded. Give an example of an unbounded continuous real valued function on $\prod X_n$ when it is endowed with the box topology. Justify your answer.

3. Define linear continuum. Let $X$ be an ordered set. If $X$ is connected with respect to the order topology, then prove that $X$ is a linear continuum.

4. Let $X$ be a connected metric space. Then prove that either $X$ is a singleton set or contains uncountable number of points. Give an example of a connected topological space with countable number of points.

5. Let $X$ and $Y$ be topological spaces and $\mathcal{A}$ be a collection of basic open sets in $X \times Y$ such that no finite subcollection of $\mathcal{A}$ covers $X \times Y$. If $X$ is compact, show that there exists a point $x \in X$ such that no finite subcollection of $\mathcal{A}$ covers $\{x\} \times Y$.

6. Define a regular topological space. Prove that if a space $X$ is regular, then any two distinct points of $X$ have open neighbourhoods whose closures are disjoint.