Answer all questions.

1. Give an example of a topological space which is 
a) Hausdorff but not regular 
b) limit point compact but not compact 
c) metrizable but not second countable 
d) second countable but not metrizable 
e) path connected but not locally connected.

Please justify each answer. [20]

2. Let $H = \prod_{n \in \mathbb{Z}_+} [0, 1/n]$ denote the Hilbert cube. Prove that the product topology and the uniform topology on $H$, inherited as a subspace of $\mathbb{R}^\omega$, are the same. [5]

3. Prove that a topological space is second countable and normal, if and only if, it can be imbedded as a subspace of the Hilbert cube (with the uniform topology). [5]

4. Define an equivalence relation on the points of the $n$-sphere, $S^n$, as follows: for $x, y \in S^n$ we have $x \sim y$, if and only if, either $y = x$ or $y = -x$. Let $p : S^n \to S^n/ \sim$ denote the quotient map. Here $S^n$ has the usual subspace topology inherited from the euclidean topology of $\mathbb{R}^{n+1}$, and $S^n/ \sim$ has the quotient topology. Is $p$ an open map? Is $p$ a closed map? [5]