Indian Statistical Institute  
M.Math I Year  
First Semester Examination, 2005-2006  
General Topology  

Time: 3 hrs Date: 25-11-05

Attempt any five questions. All questions carry equal marks. Any result proved in the class may be cited and used without proof.

1. a) Let $X$ be compact and Hausdorff, $A \subseteq X$ be closed. Show that $X/A$ is homeomorphic to the one-point compactification of $X - A$.

b) Describe explicitly the quotient topology on the quotient group $\mathbb{R}/\mathbb{Q}$, $\mathbb{R}$ being the real line, $\mathbb{Q}$ the set of rationals, treated as a subgroup of the group $(\mathbb{R}, +)$.

2. a) Prove that $GL(n, \mathbb{C})$ is path connected (hint; use the polynomial $p(z) = \det((1 - z)I + zA)$ for $A \in GL(n, \mathbb{C}))$.

b) Prove that any discrete subgroup of $S^1$ must necessarily be finite cyclic.

3. a) Let $X$ be any space. Show that $CX$, the cone over $X$ is contractible.

b) Show that $S^{n-1}$ is a deformation retract of $S^n - \{N, S\}$, $N$ and $S$ being the north and south poles of $S^n$ respectively.

4. Let $f, g : X \to S^n$ be continuous maps with $f(x) \neq -g(x) \ \forall \ x \in X$. Prove that $f \simeq g$.

5. Let $X$ be a space. Then show that $X$ is path connected if and only if all constant maps: $X \to X$ are homotopic to each other.

6. Let $R_\theta : S^1 \to S^1$ be a rotation by angle $\theta$. Show that $R_\theta$ is homotopic to the identity map: $S^1 \to S^1$.

7. Let $G$ be a connected group, $H$ a discrete normal subgroup. Prove that $H \subseteq Z(G)$, the centre of $G$. 

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