

# Combinatorics and Graph Theory

M. Math. II

Mid-Term Examination

**Instructions:** All questions carry ten marks. All graphs are assumed to be simple.

1. Prove that in any non-trivial Steiner system  $S(t, k, v)$ , we must have

$$v \geq (t + 1)(k - t + 1)$$

2. Prove that the existence of a  $(t + 1) - (v + 1, k + 1, \lambda)$  design implies the existence of a  $t - (v, k, \lambda)$  design. Give an example to show that the converse is not true.
3. Let  $G$  be a  $k$ -regular graph with  $k > 1$ . If  $G$  has a cut-edge, prove that  $k$  must be odd and it must have at least  $2k + 4$  vertices.
4. Let  $k$  be a natural number and let  $Q_k$  denote the graph whose vertices are  $k$ -tuples with entries in  $\{0, 1\}$  and edges are pairs of  $k$ -tuples that differ in exactly one position. Prove that the complete bipartite graph  $K_{2,3}$  is not a subgraph of  $Q_k$ .
5. Prove or disprove: Every graph contains at least one non cut-vertex.