Indian Statistical Institute, Bangalore
M. Math.
First Year, Second Semester
Functional Analysis

Mid-term Supplementary Examination — Date: 01 March 2024
Maximum marks: 54 — Time: 2 hours
Instructor: Chaitanya G K

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1. Let D be the open unit disc in $\mathbb{C}$ and $\overline{D}$ be its closure. Let X be the set of all complex-valued functions on $\overline{D}$ which are analytic in D. For $f \in X$, let
$$\|f\| = \sup\{|f(e^{it})| : 0 \leq t \leq 2\pi\}.$$
Show that X is a Banach space. [10]

2. Show that the dual of l^1 is l^∞. [10]

3. State and prove Riesz lemma. [12]

4. Is the Riesz representation theorem valid in general inner product spaces? Justify your answer. [8]

5. Let X be an inner product space. Prove that $x_n \to x$ if and only if $\langle x_n, y\rangle \to \langle x, y\rangle$ uniformly for y in X with $\|y\| = 1$. [10]

6. Let H be a Hilbert space and $T : H \to H$ be an isometric linear map. Show that the following statements are equivalent:
(a) T is unitary.
(b) T is onto.
(c) The range of T is dense in H. [10]

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