COMPLEX ANALYSIS MIDTERM

I have not used any unfair or illegal means to answer any of the questions in this exam.

Name: **Signature:**

You may use the theorems we have done in class for the questions without having to reprove them - but please state what you use.

Let f be a holomorphic function on $\mathbb{C}$ such that

$$|f(z)| \leq A|z|^n + B$$

for some positive real numbers A and B and for all $z \in \mathbb{C}$. Show that f is a polynomial. 10

2. Prove that the function $f(z) = \frac{1}{z}$ has a primitive in $D(1, 1)$, the open disc of radius 1 around 1. 5

3. Let f and g be holomorphic functions on $\mathbb{C}$. If there exists $a \in \mathbb{C}$ such that f_a and g_a lie in the same connected component of $\mathcal{O}$, show that $f = g$. 10

4. State Morera's Theorem. 5