Indian Statistical Institute, Bangalore

M. Math.I Year, Second Semester

Mid-Sem Examination

Complex Analysis

Time: 3 hours February 26, 2010 Instructor: Bhaskar Bagchi

Maximum Marks 100

1. Show that for $z, w \; e \; \mathbb{C}$, we have

   $$| \; (w^n - z^n) - nz^{n-1}(w - z) \; | \leq n(n-1) \; | \; z - w \; |^2 \; . \; (\max(| \; z \; |, | \; w \; |))^{n-2}$$

   for $n \geq 2$.

   Hence deduce that any power series may be differentiated term by term within its disc of convergence. [20]

2. Prove that a non-constant analytic function has no local maximum for its modulus. Deduce that all its local minima are zeros. [20]

3. Prove ab initio that there is a $\theta > 0$ such that $e^{i\theta} = 1$. [20]

4. If Ω is a convex domain then show that every holomorphic function on Ω has an anti-derivative. [20]

5. If $f : \widehat{\mathbb{C}} \longrightarrow \widehat{\mathbb{C}}$ is analytic then show that f is a rational function. [20]