

## COMPLEX ANALYSIS FINAL BACK PAPER

I have not used any unfair or illegal means to answer any of the questions in this exam.

Name:

Signature:

You may use the theorems we have done in class for the questions without having to reprove them - but **please state what you use**. For an open set  $\Omega$ ,  $\mathcal{H}(\Omega)$  denotes the set of holomorphic functions on  $\Omega$ .

1a. State the Residue Theorem. (5)

1b.. State and prove the Argument Principle. (10)

2. Evaluate

a.  $\int_0^{2\pi} \frac{dt}{2 + \sin(t)}$  (10)

b.  $\int_0^{\infty} \frac{\cos(x)dx}{(1+x^2)^3}$  (10)

3a. State and prove Schwarz's Lemma. (8)

3b. Let  $D$  be the open unit disc  $\{z \mid |z| < 1\}$ . Show that if  $f : D \rightarrow D$  is an analytic automorphism then there exists  $\theta \in \mathbb{R}$  and  $a \in D$  such that (7)

$$f(z) = e^{i\theta} \frac{z - a}{1 - \bar{a}z}$$