Indian Statistical Institute, Bangalore
M. Math.I Year, Second Semester
Semestral Examination
Complex Analysis (Back Paper)

Time: 3 hours Instructor: Bhaskar Bagchi

Maximum Marks 100

1. Define the order of an entire function. Show for each retional number $\lambda \geq 0$ there is an entire function of order λ. [20]

2. Define the Mobius group. Prove that it is the full group of bi-holomorphic automorphisms of the open unit disc. [20]

3. Let $H = \{z = x + iy : y > 0\}$ be the upper half plane and let Ω be any planar domain. Show the class of all holomorphic functions from Ω to H is pre-compact (normal) in the space of all holomorphic functions on Ω. [30]

4. Let $\{f_n\}$ be a sequence of entire functions which converges locally uniformly to an entire function f. If each f_n non-vanishing then show that f is either non-vanishing or identically zero. Show that both possibilities can occur.

   [20]

5. Use the residue theorem or the principle of analytic continuation to compute $\int\limits_{-\infty}^{\infty} e^{itx} e^{-x^2/2} dx, t \,\varepsilon\, \mathbb{R}$. [10]