COMPLEX ANALYSIS FINAL EXAM

I have not used any unfair or illegal means to answer any of the questions in this exam.

Name: **Signature:**

You may use the theorems we have done in class for the questions without having to reprove them - but **please state what you use**. For an open set Ω, $\mathcal{H}(\Omega)$ denotes the set of holomorphic functions on Ω.

1. Let $\Omega \subsetneq \mathbb{C}$ be an open set. Let f be in $\mathcal{H}(\Omega)$

a. Show that if Ω is simply connected, then f has a primitive. (5)

b. Show that if every f has a primitive, then Ω is simply connected. (10)

2. Consider the integral

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$$

The value is

(1) π

(2) $\pi/2$

(3) $3\pi/2$

(4) $\pi^2/2$

Justify your answer. (10)

3. Let Ω be a connected open set in $\mathbb{C}$. Let $\{f_n\}$ be a sequence of holomorphic functions on Ω converging uniformly on compact sets to a function f.

a. Is f holomorphic? If yes state why. If no, give a counterexample. (5)

b. Show that if f_n are nowhere zero on Ω and f is non-constant, then f is nowhere 0 on Ω. (10)

4. Let P be a *non-constant* polynomial.

a. State why $P(\mathbb{C})$ is open. (3)

b. Show that $P(\mathbb{C})$ is closed (5)

c. Conclude that P has a root in $\mathbb{C}$. (2)