Indian Statistical Institute, Bangalore
M. Math.I Year, Second Semester
Semestral Examination
Complex Analysis

Time: 3 hours May 07, 2010 Instructor: Bhaskar Bagchi

Maximum Marks 100

Remark: This paper carries a total of 110 marks. Answer as much as you can. The maximum you may score is 100.

1. If Ω is a simply connected proper sub-domain of $\mathbb{C}$, then show that there is a one-one bounded analytic function on Ω. (You may not use the Reimann mapping theorem !). [20]

2. If $f : \Omega \longrightarrow \mathbb{C}$ is a one-one analytic function then show that $f'(z) \neq 0$ for all z in Ω. Show that if $f'(z) \neq 0$ for all $z \in \Omega$ then the analytic function f must be locally one-one, but it need not be one-one on the whole of Ω. [30]

3. Let $w = e^{2\pi i/n}$ for some positive integer n. Then construct an entire function f of order one such that $f(w\ z) = f(z)$ for all $z \in \mathbb{C}$. Hence construct an entire function of order $1/n$. [30]

4. Let f be an analytic function on the closed disc with centre z_0 and radius $r > 0$. Then show that $f(z_o) = \frac{1}{2\pi} \int\limits_0^{2\pi} f(z_0 + r\ e^{i\theta})d\theta$. Hence prove that z_0 can not be a local maximum for $\mid f \mid$ unless f is a constant. [10]

5. State and prove Jensen's formula. [20]