Topology/Geometry core courses

1) **Topology I**

Topological spaces, open and closed sets, basis, closure, interior and boundary. Subspace topology, Hausdorff spaces. Continuous maps: properties and constructions; Pasting Lemma. Homeomorphisms. Product topology.

Connected, path-connected and locally connected spaces. Lindelof and Com- ¨ pact spaces, Locally compact spaces, one-point compactification and Tychonoff's theorem. Paracompactness and Partitions of unity (if time permits).

Countability and separation axioms., Urysohn embedding lemma and metrization theorem for second countable spaces. Urysohn's lemma, Tietze extension theorem and applications. Complete metric spaces. Baire Category Theorem and applications.

Quotient topology: Quotient of a space by a subspace. Group action, Orbit spaces under a group action. Examples of Topological Manifolds.

Topological groups. Examples from subgroups of GLn(R) and Gln(C).

Homotopy of maps. Homotopy of paths. Fundamental Group.

(with MMath or with Bmath + homotopy, fundamental group to be done separately)

2) **Topology II**

Review of fundamental groups, necessary introduction to free product of groups, Van Kampen's theorem. Covering spaces, lifting properties, Universal cover, classification of covering spaces, Deck transformations, properly discontinuous action, covering manifolds, examples.

Categories and functors. Simplicial homology. Singular homology groups, axiomatic properties, Mayer-Vietoris sequence, homology with coefficients, statement of universal coefficient theorem for homology, simple computation of homology groups.

CW-complexes and Cellular homology, Simplicial complex and simplicial homology as a special case of Cellular homology, Relationship between fundamental group and first homology group. Computations for projective spaces, surfaces of genus g.

(with MMath second semester)

3) **Topology III**

CW-complexes, cellular homology, comparison with singular theory, computation of homology of projective spaces.

Definition of singular cohomology, axiomatic properties, statement of universal coefficient theorem for cohomology. Betti numbers and Euler characterisitic. Cup and cap product, Poincare duality. Cross product and statement of Kunneth theorem. Degree of maps with applications to spheres.

Definition of higher homotopy groups, homotopy exact sequence of a pair. Definition of fibration, examples of fibrations, homotopy exact sequence of a fibration, its application to computation of homotopy groups. Hurewicz homomorphism, The Hurewicz theorem. The Whitehead Theorem.

(with MMath Elective)

4) **Algebraic geometry**
(Note: For students opting for 'Algebraic Geometry', a prior knowledge of 'Commutative Algebra' is desirable.)
Topics from: Polynomial rings, Hilbert Basis theorem, Noether normalisation lemma, Hilbert Nullstellensatz, Affine and Projective spaces, Affine Schemes, Elementary dimension theory, Smoothness, Curves, Divisors on curves, Bezout's theorem, Abelian differential, Riemann Roch for curves.
(with BMath/MMath Elective)

5) **Differential geometry**
Smooth manifolds: Manifolds in R n , submanifolds, manifolds with boundary. Smooth maps between manifolds. Regular values. Examples of manifolds: A) Curves and surfaces in R 2 and R 3 . B) Level surfaces in R n+1, C) Inverse image of regular values. Tangent spaces, derivatives of smooth maps, smooth vector fields, Existence of integral curves of a vector field near a point.
Geometry of curves and surfaces: Parametrized curves in R 3 , length, integral formula for smooth curves, regular curves, parametrization by arc length. Osculating plane of a space curve, Frenet frame, Frenet formula, curvature, invariance under isometry and reparametrization. Discussion of the cases for plane curves, rotation number of a closed curve, osculating circle, 'Umlaufsatz'.
Surfaces in R 3 : Existence of a normal vector of a connected surface. Gauss map. The notion of a geodesic on a surface. The existence and uniqueness of a geodesic on a surface through a given point with a given velocity vector thereof. Covariant derivative of a smooth vector field. Parallel vector field along a curve. Existence and uniqueness theorem of a parallel vector field along a curve with a given initial vector. The Weingarten map of a surface at a point, 16 its self-adjointness property. Normal curvature of a surface at a point in a given direction. Principal curvatures, first and second fundamental forms, Gauss curvature and mean curvature. Gauss-Bonnet theorem (statement only).
Differential forms and orientation: Differential Forms, Orientation of manifolds, Integration of forms, Stokes' Theorem. (proof to be given if time permits). Proof of Gauss-Bonnet theorem (if time permits).
(with MMath 3rd semester)

6) **Differential Geometry II**
Manifolds and Lie groups, Frobenius theorem, Tensors and Differential forms, Stokes theorem, Riemannian metrics, Levi-Civita connection, Curvature tensor and fundamental forms.
(with BMath 3rd year)

7) **Differential Topology**
Manifolds. Inverse function theorem and immersions, submersions, transversality, homotopy and stability, Sard's theorem and Morse functions, Embedding manifolds in Euclidean space, manifolds with boundary, intersection theory mod 2, winding numbers and Jordan- Brouwer separation theorem, Borsuk-Ulam fixed point theorem.
(with BMath 3rd year)