Algebra core courses

1) **Algebra 1** :
Group theory: Review, Group actions, Sylows' theorems.
Rings and Modules: Review (ED, PID, UFD, prime and maximal ideals and modules). Structure theorem of finitely generated modules over PID. Tensor product of modules.
Linear algebra: Review (Vector spaces, direct sums, tensor products; Linear transformations and Matrices; Determinants). Diagonalizability and Nilpotence; Rational canonical form, Jordan form. Bilinear forms; Inner product spaces; unitary, self-adjoint, normal, and isometric transformations; Spectral theorem.

2) **Algebra 2** : (can be with BMath 3rd year or MMath 2nd year)
Galois theory: Algebraic and transcendental extensions; algebraic closure; splitting fields and normal extensions; separable, inseparable and purely inseparable extensions; finite fields. Galois extensions and Galois groups, Fundamental theorem of Galois theory, cyclic extensions, solvability by radicals, constructibility of regular n-gons, cyclotomic extensions.
Representation theory: Introduction to multilinear algebra: Review of linear algebra, multilinear forms, tensor products, wedge product, Grassmann ring, symmetric product. Representation of finite groups:Complete reducibility, Schur's lemma, characters, projection formulae. Induced representation, Frobenius reciprocity. Representations of permutation groups.

3) **Commutative algebra** (same MMath 2nd year elective)
Quick review of Rings and ideals: ideals in quotient rings; prime ideals under quotient, existence of maximal ideals; operations on ideals (sum, product, quotient and radical); ideals and prime ideals in product rings, Chinese Remainder theorem; nilradical and Jacobson radical; extension and contraction of ideals under ring homomorphisms;
Free modules; Projective Modules; Tensor Product of Modules and Algebras; Flat, Faithfully Flat and Finitely Presented Modules; Shanuel's Lemma.
Localisation and local rings, universal property of localisation, extended and contracted ideals and prime ideals under localisation, localisation and quotients, exacteness property, Nagata's criterion for UFD and applications.
Prime avoidance. Results on prime ideals like theorems of Cohen and Isaac, equivalence of PID and one-dimensional UFD.
Modules over local rings. Cayley-Hamilton, NAK lemma and applications. Examples of local-global principles. Projective and locally free modules. Patching up of Localisation.
Polynomial and Power Series Rings. Noetherian Rings and Modules. Hilbert's Basis Theorem, Graded Rings, equivalence of Noetherian rings and finitely generated algebras for graded rings.
Integral Extensions: integral closure, normalisation and normal rings. CohenSeidenberg Going-Up Theorem. Hilbert's Nullstellensatz and applications. Introduction to Valuation rings.
Time permitting: Introduction to Grobner basis.

4) **Algebraic number theory** (can be with BMath 3rd year or MMath 2nd year)
Algebraic numbers and algebraic integers; Brief review of integral extensions; Norm, trace and discriminant; Existence of integral basis. Dedekind domains, ideal class group. Minkowsky theory, finiteness of class group. Dirichlet unit theorem. Factoring of prime ideals on extensions, fundamental identity; Quadratic number fields (computation of class numbers, prime decomposition, Pell's equations). Hilbert's ramification theory (decomposition and inertia groups); Cyclotomic fields. Valuations, completions, local fields.