Permutation modules and p-ranks of Incidence Matrices
Part 3: Cross-characteristic

Peter Sin

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Groups and Geometries, ISI Bangalore, December 2012

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Outline

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$\mathbf{GL}(V)$ acting on $\mathbf{P}(V)$

- V a vector space over $\mathbf{F}_q$.
- $\mathbf{GL}(V)$ acts doubly transitively on the set P of 1-dimensional subspaces of V.
- F an algebraically closed field of characteristic $\ell \nmid q$.
- F^P the FG-permutation module.
- If $\ell \nmid |P|$, $F^P = F \oplus X$

- $$\begin{array}{c} F \\ | \\ X \\ | \\ F \end{array}$$

Figure: F^P when $\ell \mid |P|$

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- Suppose V has a non-dengenerate quadratic form or symplectic form, or a vector space over $\mathbf{F}_{q^2}$ with a nonsingular hermitian form.
- G, the subgroup of $\mathbf{GL}(V)$ preserving the form.
- $\mathbf{P}_0$ the set of singular 1-spaces (points).
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Outline

Characteristic zero

- $\Delta : F^{\mathbf{P}_0} \to F^{\mathbf{P}_0}$, $x \mapsto \sum_{(x,y)\in\Psi} y$
- $F^{\mathbf{P}_0} = \mathbf{F1} \oplus X \oplus Y$
- D. G. Higman (1960s)
- The summands are the eigenspaces Δ.
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- Liebeck (1980-81) studied $F^{\mathbf{P}_0}$ under the assumption $c \neq d$
- *graph submodules* U'_c, U'_d, where $U'_\lambda = \langle (\Delta - \lambda I)(x - x') \mid, x, x' \in X \rangle$
- Found the submodule structures of F^X in these cases.

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Structure of $F^{\mathbf{P}_0}$ when $c \neq d$

$a \notin \{c, d\}$:

$$F \oplus X \oplus Y$$

$a \in \{c, d\}$:

$$X \oplus \begin{array}{c} F \\ | \\ Y \\ | \\ F \end{array}$$

Figure: The cases $c \neq d$

Outline

$F^{\mathbf{P}_0}$ when $c = d$

- **Sp**$(2m, q)$ q odd, $\ell = 2$ (Lataille-Sin-Tiep (2003))
  - m even :

m odd :

Figure: **Sp**$(2m, q)$, q odd, $\ell = 2$

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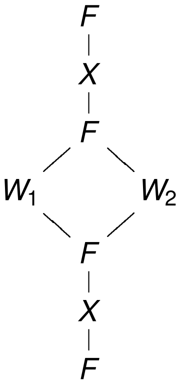

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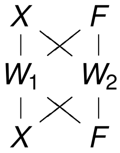

Figure: $\mathbf{Sp}(2m, q)$, q odd, $\ell = 2$

Structure of module of lines for $m = 2$, $\ell = 2$

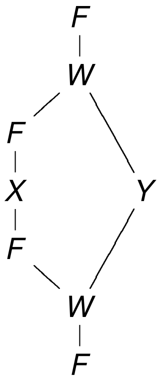

Figure: lines for $\mathbf{Sp}(4, q)$, q odd, $\ell = 2$

Related work on GQ codes

The $\mathbf{F}_2$-permutation modules for rank 2 groups of odd characteristic have been studied in small ranks by Bagchi-Brouwer-Wilbrink (1991), and Brouwer-Haemers-Wilbrink (1992) in connection with the $\mathbf{F}_2$-codes associated with generalized quadrangles.

$F^{\mathbf{P}_0}$, remaining $c = d$ cases

- Sin-Tiep (2005)
- $\mathbf{GU}(2m, q^2)$ with $m \geq 2$ and $\ell | (q+1)$
- $\mathbf{GU}(2m+1, q^2)$ when $m \geq 2$, $\ell | (q+1)$
- $\mathbf{GO}(2m+1, q)$ with $m \geq 3$, q odd and $\ell = 2$
- $\mathbf{GO}^{+}(2m, q)$ with $m \geq 3$ and $\ell | (q+1)$
- $\mathbf{GO}^{-}(2m, q)$ with $m \geq 3$ and $\ell | (q+1)$

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Unitary groups in even dimension

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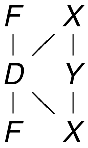

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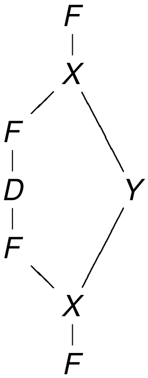

Figure: $F^{\mathbf{P}_0}$ for $\mathbf{GU}(2m, q^2)$ when $\ell \mid (q+1)$.

Unitary groups in odd dimension

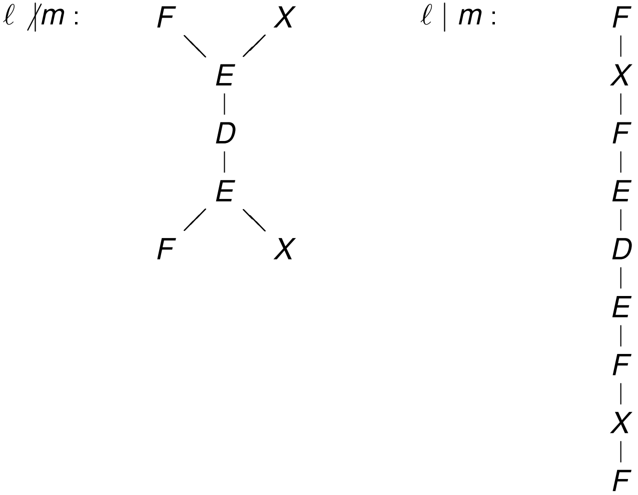

Figure: Submodule structure of $F^{\mathbf{P}_0}$ for $\mathbf{GU}(2m+1, q^2)$ when $\ell \mid (q+1)$ and ℓ is odd or $\ell = 2$ and $q \equiv 3 \pmod 4$.

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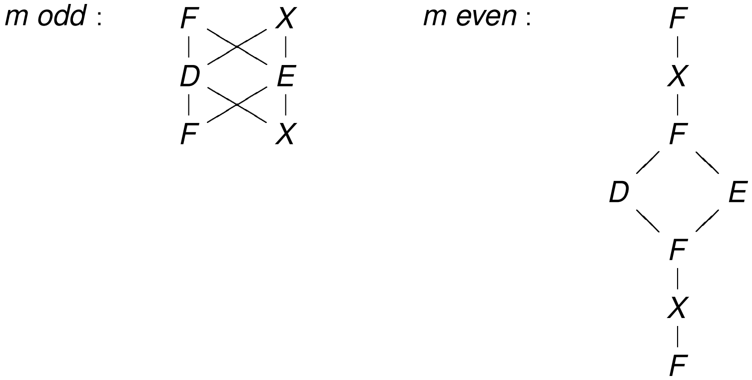

Figure: Submodule structure of $F^{\mathbf{P}_0}$ for $\mathbf{GU}(2m+1, q^2)$ when $\ell = 2$ and $q \equiv 1 (\text{mod } 4)$.

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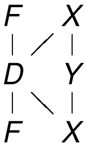

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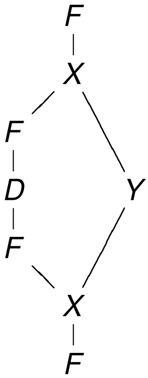
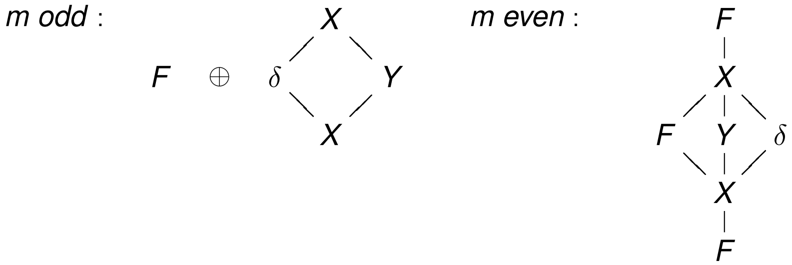


Figure: Submodule structure of $F^{\mathbf{P}_0}$ for $\mathbf{GO}(2m+1, q)$, q odd, when $\ell = 2$.

Orthogonal groups in even dimension, maximal index



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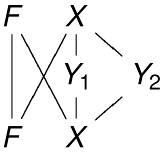

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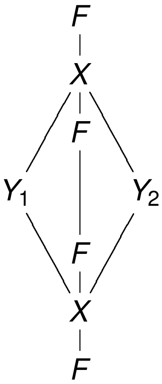

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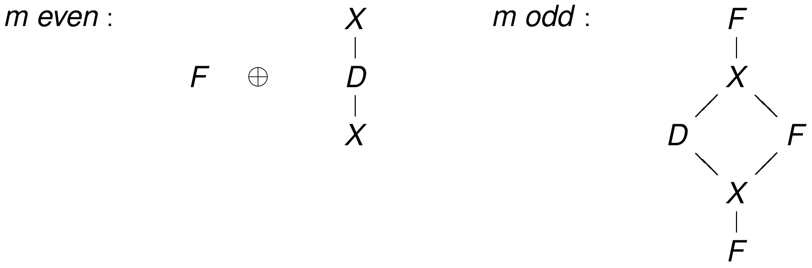

Figure: Submodule structure of $F^{\mathbf{P}_0}$ for $\mathbf{GO}^{-}(2m, q)$ when $\ell \neq 2$ and $\ell \mid (q+1)$.

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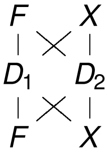

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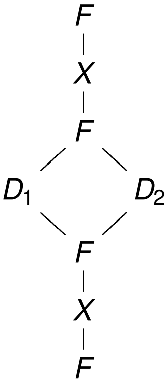

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Further work

- Hall-Nguyen, rank 3 permutation modules on nonsingular points, $O_{2m}^{\pm}(2)$, $m \geq 2$ and $U_m(2)$, $m \geq 4$.
- There are two rank 3 permutation modules for $E_6(q)$, related by an automorphism.

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Outline

Coding theory examples

The cross-characteristic theory, in particular $\ell = 2$, shows up in coding theory, in connection with structured Low Density Parity Check (LDPC) Codes. These may use the $\mathbf{F}_2$-incidence matrices of a family of geometrically defined incidence relations as generator or parity-check matrices.

$LU(3, q)$ codes

- V a 4-dimensional vector space over the field $\mathbf{F}_q$
- Assume V has a nonsingular alternating bilinear form.
- $P = \mathbf{P}(V)$, $L =$ the set of totally isotropic 2-dimensional subspaces , lines in P.
- Fix a point p_0 and a line ℓ_0 through p_0.
- $P_1 = P \setminus p_0^{\perp}$,
- $L_1 =$ set of lines that do not meet ℓ_0.
- Consider the incidence systems (P_1, L_1),
- $M(P, L)$, $M(P_1, L_1)$ incidence matrices with $\mathbf{F}_2$ entries.

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- $M(P,L)$, $M(P_1, L_1)$ incidence matrices with $\mathbf{F}_2$ entries.

$LU(3,q)$ codes

- $LU(3,q)$ codes defined by Kim, Peled, Perepelitsa, Pless, Friedland (2004)
- P^* and L^* sets in bijection with $\mathbf{F}_q{}^3$
- $(a,b,c) \in P^*$ is incident with $[x,y,z] \in L^*$ iff

$$y = ax + b \qquad \text{and} \qquad z = ay + c.$$

- The $LU(3,q)$ codes are defined using the incidence matrix and its transpose as parity check matrices.
- Kim et. al. gave a conjecture for dim $LU(3,q)$, q odd.
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- If q is even, $\mathrm{rank}_2\, M(P,L) = 1 + (\frac{1+\sqrt{17}}{2})^{2t} + (\frac{1-\sqrt{17}}{2})^{2t}$. (Sastry-Sin)
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Codes from a conic

- $PG(2, q)$, q odd.
- $\mathcal{O}$ conic.
- Points: $\mathcal{O}$, E (external) I (internal)
- Lines: Ta (tangent) , Se (secant), Pa (passant)
- Droms and Mellinger used the various point-line incidence matrices to define families of LDPC codes.
- Conjectures for dimensions based on computer calculations.

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Conjectures of Droms and Mellinger

- $$\mathrm{rank}_2 A(E, Se) = \begin{cases} \frac{1}{4}(q-1)^2 + 1, & \text{if } q \equiv 1 \pmod 4, \\ \frac{1}{4}(q-1)^2 - 1, & \text{if } q \equiv 3 \pmod 4. \end{cases}$$
- Sin-Xiang-Wu (2011) gave a proof.
- Proof uses detailed information about 2-blocks of $\mathbf{SL}(2, q)$ (Landrock 1980).
- Wu has recently solved the corresponding 2-rank conjectures of Droms and Mellinger for the other possible incidences of points and lines.

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