

# Academic proposal for AIS in Representation Theory

Name of the school, venue, and dates

AIS in Representation Theory  
ISI Bangalore Centre  
02–24 June 2010

Names of organizers

K. N. Raghavan Matscience, Chennai	N. S. Narasimha Sastry ISI, Bangalore
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Names of resource persons

Bhaskar Bagchi (ISI, Bangalore)  
Shripad Garge (IIT, Bombay)  
Upendra Kulkarni (ISI, Bangalore)  
Amit Kulshrestha (IISER, Mohali)  
K. N. Raghavan (Matscience, Chennai)  
N. S. Narasimha Sastry (ISI, Bangalore)  
Anupam Kumar Singh (IISER, Pune)  
Maneesh Thakur (ISI, Delhi)

## A brief description of the school for publicity

The REPRESENTATION THEORY OF FINITE GROUPS has been an area of intense research ever since its birth in the late nineteenth century. The theory is an important tool in several branches of mathematics, e.g., geometry, number theory, and combinatorics. Moreover, the unsolved mysteries and conjectures intrinsic to the theory, like the conjectures of Lusztig, Alperin, and Dade, are expected to keep it an active area of research.

This AIS is aimed at those pursuing (or intending to pursue) research in any related area of mathematics (interpreted in a broad sense). The basic concepts and results of both the ordinary and the modular theory (particularly the latter) will be introduced. A basic knowledge of mathematics (especially of algebra) as taught in the *foundational schools* will be assumed but no more.

Those working with or expecting to work with groups of any kind whatsoever (finite, Lie, algebraic,  $p$ -adic, quantum, ...) are encouraged to apply. Students with the pre-requisite background but still undecided about their area of research are also encouraged to apply. Participants apart, post-docs and young faculty working in the area will be invited to participate (to be in charge of tutorials or just to attend).

# Syllabus

Speaker	No. of lectures	Syllabus
I (BB)	3	Linear representations of groups; modules over group rings; indecomposable and irreducible representations; Schur's lemma and complete reducibility; constructing new representations from given ones: tensor, symmetric and exterior products; duals. Induced representations; Frobenius reciprocity; Mackey's criterion for irreducibility.
II (SG)	3	Radicals of modules and rings; Artin-Wedderburn structure theory of semi-simple rings and modules over them; applications to group rings of finite groups.
III (BB)	3	Ordinary theory: characters; orthogonality and other properties of irreducible characters. Burnside's $p^a q^b$ theorem.
IV (UK)	2	Artin-Brauer characterization of a character; Clifford theory.
V (AKS)	3	See (1) and (3) of note below
VI (AK)	3	See (2) and (4) of note below.
VII (NSNS)	3	p-adic numbers and p-adic integers; p-modular system for a group; lifting of idempotents, projective and free modules;
VIII (KNR)	5	Cartan-Brauer triangle and its properties; Brauer characters and their orthogonality.
IX (KNR)	5	Vertices and sources; trivial intersection, Green correspondence.
X (NSNS)	7	Blocks and defect groups; Brauer correspondence; Brauer's theorems and Alperin's conjecture.

## Resource persons

Bhaskar Bagchi (ISI, Bangalore) (BB), Shripad Garge (IIT, Bombay) (SG), Upendra Kulkarni (ISI, Bangalore) (UK), Amit Kulshrestha (IISER, Mohali) (AK), K. N. Raghavan (Matscience, Chennai) (KNR), N. S. Narasimha Sastry (ISI, Bangalore) (NSNS), Anupam Kumar Singh (IISER, Pune) (AS), Maneesh Thakur (ISI, Delhi) (MT)

## Note

- (1) Quick review of: Noetherian and Artinian rings and modules; existence of composition series of modules. Projective, injective, relatively projective and relatively injective modules  
Dornhoff § 39 and § 51
- (2)  $I$ -adic topologies of rings and modules, completeness of modules relative to  $I$ -adic topologies  
Dornhoff § 42, Theorems 42.11 and 42.14  
See Feit Chap I § 9, Theorems 9.3, 9.10, 9.11
- (3) Local rings, unique decomposition theorem, Krull-Schmidt theorem.

Feit, Chap. I § 10, 11  
Dornhoff § 43.

- (4) Finite extensions of complete local domains  
Dornhoff § 49 Theorems 49.4, 49.10.  
See also Feit, Chap I, § 18

## Timetable

Day	Date	Lecture 1 0900–1030	Lecture 2 1100–1230	Tutorial 1 1530 to 1630	Tutorial 2 1645–1745
1	2nd	I	V	I	V
2	3rd	I	V	I	V
3	4th	I	V	I	V
4	5th	III	II	III	II
5	7th	III	II	III	II
6	8th	III	II	III	II
7	9th	III	VI	III	VI
8	10th	IV	VI	IV	VI
9	11th	IV	VI	II	VI
10	12th	VII	VII (Tutorial) 1100–1200		
11	14th	VII	VIII	VII	VIII
12	15th	VII	VIII	VII	VIII
13	16th	X	VIII	X	VIII
14	17th	X	VIII	X	VIII
15	18th	X	VIII	X	VIII
16	19th	IX	IX (Tutorial) 1100-1200		
17	21st	X	IX	X	IX
18	22nd	X	IX	X	IX
19	23rd	X	IX	X	IX
20	24th	X	IX	X	IX

The UNITY OF MATHEMATICS lectures will be scheduled during 1400–1500 on some three or four days of the School.

## Contact details of organizers

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