# Semi-regular maps on surfaces

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Celebration of Bhaskar Bagchi's career at ISI Indian Statistical Institute Bangalore

January 29, 2019

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Speaker with Bhaskar Bagchi at Oberwolfach in February 2015

# Outline of the talk

1. Examples: Lots of symmetrical geometric objects/toys are here.

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- 2. Definitions: Some necessary definitions are here.
- 3. Results: Here I present some results related to my works.
- 4. Open Problems: Some open problems are stated here.
- 5. References: Few references are here.
- 6. Acknowledgements:

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### Five Platonic Solids



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#### Platonic Solids

The tetrahedron, cube and octahedron occur in nature as crystals (of various substances, such as sodium salphantimoniate, common salt, and chrome alum, respectively). The dodecahedron, icosahedron occur as skeletons of microscopic sea animals called radiolaria, the most perfect examples being *Circogonia icosahedra* and *Circorrhegma dodecahedra*.

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Circogonia icosahedra, Circorrhegma dodecahedra

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Circogonia icosahedra, Circorrhegma dodecahedra Excavations on Monte Loffa, near Padua, have revealed an Etruscan dodecahedron which shows that this figure was enjoyed as a toy at least 2500 years ago. These were built up by Plato (about 400 B.C) and before him by the earliest Pythagoreans.

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#### Archimedean Solids

The Archimedean solids take their name from Archimedes, who discussed them in a now-lost work. Pappus refers to it, stating that Archimedes listed 13 polyhedra. During the Renaissance, artists and mathematicians valued pure forms with high symmetry, and by around 1620 Kepler had completed the rediscovery of the 13 polyhedra, as well as defining the prisms, antiprisms.

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In geometry, Archimedean solids are the semi-regular convex polyhedra composed of regular polygons meeting in identical vertices, excluding the 5 Platonic solids and excluding the prisms and antiprisms.

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In geometry, Archimedean solids are the semi-regular convex polyhedra composed of regular polygons meeting in identical vertices, excluding the 5 Platonic solids and excluding the prisms and antiprisms.

"Identical vertices" means that each pair of vertices are symmetric to each other: A global isometry of the entire solid takes one vertex to the other while laying the solid directly on its initial position. Such a solid is now called a vertex-transitive solid. We will discuss this later.

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#### Prisms/Drums, Antiprisms

#### Two series of vertex-transitive maps on the 2-sphere.



Triangular-, Hexagonal-, Octagonal-, Decagonal-prism

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#### Prisms/Drums, Antiprisms

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Triangular-, Hexagonal-, Octagonal-, Decagonal-prism



Triangular-, Pentagonal-, Octagonal-, Octagonal-antiprism

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#### Vertex-transitive Tilings on Torus

Torus is one object known to all. We also know various regular tilings on the torus. Here we consider few. We will discuss more on this later.



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#### Eight non-regular Archimedean tilings on the plane

We all like symmetrical tilings. The following eight are such tilings on the plane. These are vertex-transitive. Grünbaum (1987) calls such tilings Archimedean in parallel to the Archimedean solids. There are 3 more. We will discuss these later.



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#### Uniform Tilings on the Hyperbolic Plane

In hyperbolic geometry, a uniform hyperbolic tiling is an edge-to-edge filling of the hyperbolic plane/disk which has regular polygons as faces and is vertex-transitive. It follows that all vertices are congruent, and the tiling has a high degree of rotational and translational symmetry. Some examples are given here.



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#### Some more Uniform Tiling on the Hyperbolic Plane



 $\text{Maps of types } [3^4,7], \, [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,7], \, [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,7], \, [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,7], \, [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,7], \, [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,8], \, [4,8,10], \, [4,5,4,5], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,8], \, [4,8,10], \, [4,8,10], \, [4,8,16], \, [4,8,16], \, [4,10,12], \, [5,6,5,6], \, [3,4,7,4] \\ \text{Maps of types } [3^4,8], \, [4,8,10], \, [4,8,10], \, [4,8,16], \, [4,8$ 

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#### Rhombicuboctahedron, Pseudo-rhombicuboctahedron

Grünbaum (2009) observed that a 14th polyhedron, the pseudo-rhombicuboctahedron, meets a weaker definition (namely, **semi-regular**) of an Archimedean solid, in which "identical vertices" means merely that the faces surrounding each vertex are of the same types.



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Rhombicuboctahedron and Pseudo-rhombicuboctahedron

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Rhombicuboctahedron and Pseudo-rhombicuboctahedron

Grünbaum pointed out a frequent error in which authors define Archimedean solids using this local definition but omit the 14th polyhedron. If only 13 polyhedra are to be listed, the definition must use global symmetries of the polyhedron rather than local neighborhoods.

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# Maps or Tiling on a Surface

It is now time to give some definitions.

# **Definition** (Map/Tiling)

If a (simple) graph G is embedded on a surface S then the closure of the components of  $S \setminus G$  are called *faces*. If all the faces are 2-disks and intersection of any two intersecting faces is either a vertex or an edge of G then G together with the collection of faces is called a *map* or *tiling* on S. Vertices (resp. edges) of G are called *vertices* (resp. *edges*) of the map.

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#### Semi-regular and vertex-transitive maps

# **Definition** (Type of a vertex of a map)

For a vertex u in a map X, the faces containing u form a cycle (called the *face-cycle* at u)  $C_u$  in the dual graph of M. By grouping neighbouring polygons with the same number of vertices, the cycle  $C_u$  can be decomposed in the form  $P_1 \cdots P_k \cdot F_{1,1}$ , where  $P_i = F_{i,1} \cdots F_{i,n_i}$  is a path consisting of  $p_i$ -gons,  $p_i \neq p_{i+1}$  (addition is mod k). We say that u is of  $type [p_1^{n_1}, \ldots, p_k^{n_k}]$ .

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# **Definition** (Semi-regular map)

A map X is called *semi-regular* (or *semi-equivelar*) if types of all the vertices are same. If  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  is the type of all the vertices then we say X is of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ .

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# **Definition** (Vertex-transitive map)

A map X is called *vertex-transitive* if the automorphism group of Xacts transitively on the set of vertices of X. <ロト < 母 ト < 差 ト < 差 ト 差 の Q (P) 15 /



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### Semi-regular maps on the 2-sphere

Clearly, a vertex-transitive map is semi-regular. In general the converse is not true.



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All vertex-transitive maps on the 2-sphere  $\mathbb{S}^2$  are known. These are the boundaries of Platonic solids, Archimedean solids and two infinite families of polytopes (prisms and anti-prisms) ([10], [19]). Other than these there exists a non vertex-transitive semi-regular map on  $\mathbb{S}^2$ , namely the boundary of pseudo-rhombicuboctahedron ([18]).

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## Theorem

Let X be a semi-regular map on  $\mathbb{S}^2$ . Then, up to isomorphism, X is the boundary of a Platonic solid, the boundary of an Archimedean solid, the boundary of pseudo-rhombicuboctahedron, a map of type  $[4^2, r^1]$  for some r = 3 or  $\geq 5$  (boundary of a prism) or a map of type  $[3^3, s^1]$  for some  $s \ge 4$  (boundary of an anti-prism).

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## Semi-regular maps on the real projective plane $\mathbb{RP}^2$

It is known that quotients of ten centrally symmetric vertex-transitive maps on  $\mathbb{S}^2$  (namely, the boundaries (i) icosahedron, (ii) dodecahedron, (iii) truncated octahedron, (iv) icosidodecahedron, (v) small rhombicuboctahedron, (vi) great rhombicuboctahedron, (vii) small rhombicosidodecahedron, (viii) great rhombicosidodecahedron, (ix) truncated dodecahedron and (x) truncated icosahedron) are all the vertex-transitive maps on  $\mathbb{RP}^2$  ([10]).

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## Theorem

Let Y be a semi-regular map on  $\mathbb{RP}^2$ . Then, the type of Y is [5<sup>3</sup>], [3<sup>5</sup>], [4<sup>1</sup>, 6<sup>2</sup>], [3<sup>1</sup>, 5<sup>1</sup>, 3<sup>1</sup>, 5<sup>1</sup>], [3<sup>1</sup>, 4<sup>3</sup>], [4<sup>1</sup>, 6<sup>1</sup>, 8<sup>1</sup>], [3<sup>1</sup>, 4<sup>1</sup>, 5<sup>1</sup>, 4<sup>1</sup>], [4<sup>1</sup>, 6<sup>1</sup>, 10<sup>1</sup>], [3<sup>1</sup>, 10<sup>2</sup>] or [5<sup>1</sup>, 6<sup>2</sup>] and in each case, there exists a unique map on  $\mathbb{RP}^2$  up to isomorphism. In particular, Y is vertex-transitive.

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#### Eleven Archimedean tilings on the plane



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#### Uniqueness of Archimedean tilings

There are infinitely many semi-regular map on the plane  $\mathbb{R}^2$ . If X is a semi-regular map on  $\mathbb{R}^2$  of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  then  $\sum_{i=1}^k \frac{n_i(p_i-2)}{n_i} \geq 2$ . If X is an Archimedean tiling then  $\sum_{i=1}^{k} \frac{n_i(p_i-2)}{p_i} = 2$ . We prove ([4], [5]):



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## Theorem

Let X be a semi-regular map on a surface of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ . If  $\sum_{i=1}^{k} \frac{n_i(p_i-2)}{n_i} = 2 \ then \ [p_1^{n_1}, \dots, p_k^{n_k}] = [3^6], \ [6^3], \ [4^4], \ [3^4, 6^1], \ [3^3, 4^2],$  $[3^2, 4^1, 3^1, 4^1], [3^1, 6^1, 3^1, 6^1], [3^1, 4^1, 6^1, 4^1], [3^1, 12^2], [4^1, 8^2]$  or  $[4^1, 6^1, 12^1].$ 

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# Theorem

Let X be a semi-regular map on a surface of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ . If  $\sum_{i=1}^k \frac{n_i(p_i-2)}{p_i} = 2$  then  $[p_1^{n_1}, \ldots, p_k^{n_k}] = [3^6]$ ,  $[6^3]$ ,  $[4^4]$ ,  $[3^4, 6^1]$ ,  $[3^3, 4^2]$ ,  $[3^2, 4^1, 3^1, 4^1]$ ,  $[3^1, 6^1, 3^1, 6^1]$ ,  $[3^1, 4^1, 6^1, 4^1]$ ,  $[3^1, 12^2]$ ,  $[4^1, 8^2]$  or  $[4^1, 6^1, 12^1]$ .

#### Theorem

Let X be a semi-regular map on the plane  $\mathbb{R}^2$  of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ . If  $\sum_{i=1}^k \frac{n_i(p_i-2)}{p_i} = 2$  then X is (isomorphic to) an Archimedean tiling. In particular, X is vertex-transitive.

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#### Semi-regular Tilings on Torus

Let X is a semi-regular map on the torus of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ . Since Euler characteristic of the torus is 0, it follows that  $\sum_{i=1}^k \frac{n_i(p_i-2)}{p_i} = 2$ . Thus,  $[p_1^{n_1}, \ldots, p_k^{n_k}] = [3^6]$ ,  $[6^3]$ ,  $[4^4]$ ,  $[3^4, 6^1]$ ,  $[3^3, 4^2]$ ,  $[3^2, 4^1, 3^1, 4^1]$ ,  $[3^1, 6^1, 3^1, 6^1]$ ,  $[3^1, 4^1, 6^1, 4^1]$ ,  $[3^1, 12^2]$ ,  $[4^1, 8^2]$  or  $[4^1, 6^1, 12^1]$ .



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It is known that for each of the 11 types, there exists a vertex-transitive map of that type on the torus. In particular, there are exactly 11 types of semi-regular maps on the torus. It is also know that for each types, there are infinitely many maps on the torus ([7], [8], [10], [11], [23]).

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#### Semi-regular Tilings on Torus

Let X is a semi-regular map on the torus of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ . Since Euler characteristic of the torus is 0, it follows that  $\sum_{i=1}^{k} \frac{n_i(p_i-2)}{p_i} = 2$ . Thus,  $[p_1^{n_1}, \ldots, p_k^{n_k}] = [3^6], [6^3], [4^4], [3^4, 6^1], [3^3, 4^2], [3^2, 4^1, 3^1, 4^1],$  $[3^1, 6^1, 3^1, 6^1], [3^1, 4^1, 6^1, 4^1], [3^1, 12^2], [4^1, 8^2] \text{ or } [4^1, 6^1, 12^1].$ 

It is known that for each of the 11 types, there exists a vertex-transitive map of that type on the torus. In particular, there are exactly 11 types of semi-regular maps on the torus. It is also know that for each types, there are infinitely many maps on the torus ([7], [8], [10], [11], [23]).

We have seen six vertex-transitive maps on the torus. Now we present examples of the other five different types of semi-regular maps on the torus.

Semi-regular maps on surfaces

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### Semi-regular Tilings on Torus



Maps of types  $[3^3, 4^2]$ ,  $[3^4, 6]$ , [3, 4, 6, 4], [3, 6, 3, 6],  $[3^2, 4, 3, 4]$ 

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All the known semi-regular maps on the torus are quotients of Archimedean tilings. For some types, all the known semi-regular maps are vertex-transitive. We prove ([4], [5], [8]):

#### Theorem

All semi-regular maps on the torus and the Klein bottle are quotients of Archimedean tilings.



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## Theorem

Let X be a semi-regular map on the torus. If the type of X is  $[3^6]$ ,  $[6^3]$ ,  $[4^4]$  or  $[3^3, 4^2]$  then X is vertex-transitive.

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# Theorem

If  $[p_1^{n_1}, \ldots, p_L^{n_k}] = [3^2, 4^1, 3^1, 4^1], [3^4, 6^1], [3^1, 6^1, 3^1, 6^1], [3^1, 4^1, 6^1, 4^1],$  $[3^1, 12^2], [4^1, 8^2]$  or  $[4^1, 6^1, 12^1]$  then there exists a semi-regular map of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  on the torus which is not vertex-transitive.



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Thus, there are examples (infinitely many) of semi-regular maps on the torus of seven types in which vertices form more than one orbits under the automorphism group. In [5], we prove



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# Theorem

Let X be a semi-regular map on the torus. Let the vertices of X form  $m \operatorname{Aut}(X)$ -orbits. (a) If the type of X is  $[3^2, 4^1, 3^1, 4^1]$  or  $[4^1, 8^2]$  then  $m \leq 2$ . (b) If the type of X is  $[3^4, 6^1]$ ,  $[3^1, 6^1, 3^1, 6^1]$ ,  $[3^1, 4^1, 6^1, 4^1]$  or  $[3^1, 12^2]$  then  $m \leq 3$ . (c) If the type of X is  $[4^1, 6^1, 12^1]$  then  $m \leq 6$ .

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#### Theorem

(a) If  $[p_1^{n_1}, \ldots, p_k^{n_k}] = [3^4, 6^1]$ ,  $[3^1, 6^1, 3^1, 6^1]$ ,  $[3^1, 4^1, 6^1, 4^1]$  or  $[3^1, 12^2]$ then there exists a semi-regular map M of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  on the torus with three Aut(M)-orbits of vertices. (b) There exists a semi-regular map N of type  $[4^1, 6^1, 12^1]$  on the torus with exactly six Aut(N)-orbits of vertices.

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#### Semi-regular maps on the Klein Bottle

## For Klein bottle the story is simple. In [4], we prove the following:

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Examples	Definitions	Results	Open Problems	References	Acknowledgements
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#### Semi-regular maps on the Klein Bottle

For Klein bottle the story is simple. In [4], we prove the following:

#### Theorem

Let X be a semi-regular map on the Klein bottle. (a) Then the type of X is  $[3^6]$ ,  $[6^3]$ ,  $[4^4]$ ,  $[3^3, 4^2]$ ,  $[3^2, 4^1, 3^1, 4^1]$ ,  $[3^1, 6^1, 3^1, 6^1]$ ,  $[3^1, 4^1, 6^1, 4^1]$ ,  $[3^1, 12^2]$ ,  $[4^1, 8^2]$  or  $[4^1, 6^1, 12^1]$ . (b) In each of the ten types there exists a semi-regular map on the Klein bottle which is not vertex-transitive.



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#### Semi-regular maps on negatively curved surfaces

Let X be a semi-regular map of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  on a closed surface S. Then the Euler characteristic  $\chi(S)$  of S is > 0 (resp. = 0, < 0) if and only if  $\sum_{i=1}^k \frac{n_i(p_i-2)}{p_i} < 2$  (resp. = 2, > 2). We know about such maps X. We also know maps X where  $\sum_{i=1}^k \frac{n_i(p_i-2)}{p_i} = 2$  and  $S = \mathbb{R}^2$ .

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#### Theorem

Let S be a closed surface with  $\chi(S) < 0$ . Then the number of semi-regular maps on S is at most  $-84\chi(S)$ .

Semi-regular maps on surfaces

Examples Definitions Results Open Problems References Acknowledgements

### Semi-regular Tiling on Hyperbolic Plane

Earlier we have seen 16 types of vertex-transitive hyperbolic tilings. Let us consider 8 more here.



Maps of types  $[3^2, 4, 3, 5]$ ,  $[3^2, 5, 3, 6]$ ,  $[3^2, 6, 3, 6]$ ,  $[3^2, 7, 3, 7]$ ,  $[3^3, 4, 3, 4]$ ,  $[3^5, 4]$ ,  $[4^3, 6]$ 

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Examples	Definitions	Results	Open Problems	References	Acknowledgements
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There are infinitely many types of hyperbolic tilings on the plane. In fact, there exists semi-regular map of type  $[p^q]$  on the plane whenever  $\frac{q(p-2)}{p} > 2$  ([13], [15]). Let us first state some recent results of ours.



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# Theorem ([3])

There exists a hyperbolic tiling of type [p,q,r] if and only if one of the following hold:

(i) 
$$p = q = r \ge 7$$
, or (ii)  $p$  is even,  $p \ne r$ , and  $\frac{2}{p} + \frac{1}{r} < \frac{1}{2}$ .

# Theorem (In preparation)

There exists a hyperbolic tiling of type [3, p, q, r] if and only if one of the following hold:

(i)  $q \neq p = r > 3$ , (ii) q = 3 and p = r > 6, or (iii)  $p = q = r \ge 5$ .

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In [3], we are able to construct several hyperbolic tilings under some extra hypothesis.



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In [3], we are able to construct several hyperbolic tilings under some extra hypothesis.

# Theorem

There exists a hyperbolic tiling of type  $[p_1, p_2, ..., p_d]$  if (i)  $\sum_{i=1}^d \frac{p_i-2}{p_i} > 2$ , (ii) xy appears in  $[p_1, ..., p_d]$  implies yx appears in  $[p_1, ..., p_d]$ , (iii) xy and yz appear in  $[p_1, ..., p_d]$  implies xyz appears in  $[p_1, ..., p_d]$ , (iv)  $d \ge 4$ , and each  $p_i \ge 4$ .

# Theorem

There exists a hyperbolic tiling of type  $[p_1, p_2, ..., p_d]$  if (i)  $\sum_{i=1}^{d} \frac{p_i-2}{p_i} > 2$ , (ii) xy appears in  $[p_1, ..., p_d]$  implies yx appears in  $[p_1, ..., p_d]$ , (iii) xy and yz appear in  $[p_1, ..., p_d]$  implies xyz appears in  $[p_1, ..., p_d]$ , (iv) x3y and 3yz appear in  $[p_1, ..., p_d]$  implies x3yz appears in  $[p_1, ..., p_d]$ , (v)  $d \ge 6$ .

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Examples	Definitions	Results	Open Problems	References	Acknowledgements
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We have also shown, in the previous two theorems, that if any two consecutive elements  $[p_1, p_2, ..., p_d]$  uniquely determine  $[p_1, p_2, ..., p_d]$ then the map is unique. Observe that  $[4^3, 6]$  does not satisfy this. And we have more example for this. As a corollary we get

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# Corollary

A semi-regular tiling of the hyperbolic plane of type  $[p^q]$  is unique, that is, any pair of such tilings are related by an orientation-preserving isometry of the hyperbolic plane, that takes vertices and edges of one to those of the other.

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#### A Technical Lemma

In [4], we prove this technical lemma. This is used to find necessary conditions on  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  for the existence of semi-regular maps of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ .

**Lemma:** If  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  satisfies any of the following three properties then  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  can not be the type of any semi-equivelar map on a surface.

- (i) There exists i such that  $n_i = 2$ ,  $p_i$  is odd and  $p_j \neq p_i$  for all  $j \neq i$ .
- (ii) There exists i such that  $n_i = 1$ ,  $p_i$  is odd,  $p_j \neq p_i$  for all  $j \neq i$  and  $p_{i-1} \neq p_{i+1}$ .
- (iii) There exists i such that  $n_i = 1$ ,  $p_i$  is odd,  $p_{i-1} \neq p_j$  for all  $j \neq i-1$  and  $p_{i+1} \neq p_\ell$  for all  $\ell \neq i+1$ .

(Here, addition in the subscripts are modulo k.)

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#### Some open problems

**Problem 1:** Classify all  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  such that there exists a semi-regular hyperbolic tiling of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ .

**Conjecture 1:** There exists a semi-regular hyperbolic tiling of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  if and only if  $\sum_{i=1}^k \frac{n_i(p_i-2)}{p_i} > 2$  and  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  does not satisfy any of the three properties in the previous lemma.



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**Problem 2:** Classify all  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  such that there exists a vertex-transitive hyperbolic tiling of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ .

**Conjecture 2:** For each  $[p_1^{n_1}, \ldots, p_k^{n_k}]$  with  $\sum_{i=1}^k \frac{n_i(p_i-2)}{n_i} > 2$ , there exists a unique vertex-transitive hyperbolic tiling of type  $[p_1^{n_1}, \ldots, p_k^{n_k}]$ .

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**Problem 3:** Similar questions for each closed surface of negative Euler characteristic.

I do not have any conjecture for this problem!

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# Acknowledgements

- Most of the pictures are taken from different webpages.
- The speaker thanks appropriate sites and persons for these pictures.

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This presentation is not for circulation.

# Thank You