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Binary and Grayscale Granulometries

Binary and Grayscale Granulometrie

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Mathematical Morphology (MM)

- Mathematical morphologic transformations (Matheron, 1975; Serra, 1982) have shown its speciality and strength in the context of geomorphology such as significant geomorphologic features extraction, basic measures of water bodies estimation, geomorphic processes modelling and simulation, fractal landscapes generation, etc.
- In the entire investigation, both DEM’s are analysed as grey-scale image (3-D) and the extracted networks as thresholded sets (binary form).
- In order to process the binary sets such as channel networks, binary morphological transformations are employed.
- Grey-scale mathematical morphological transformations are used to process the three-dimensional images such as DEM’s.
- The geometrical and topological structures of DEM are examined by matching it with structuring elements of various shapes and sizes at different locations in the DEM.
- Figure below provides two examples of structuring elements (*B*), which are in the shape of rhombus and square of size 3X3. (1’s and 0’s stand for foreground and background regions, respectively).

	1	
1	1	1
	1	

Rhombus

1	1	1
1	1	1
1	1	1

Square

Mathematical Morphology (cont)

<u>Binary MM</u>

- Binary erosion transformation of *S* by structuring element, *B*
  - the set of points *s* such that the translated *Bs* is contained in the original set *S*, and is equivalent to intersection of all the translates.
  - $S \ominus B = \{s: B_s \subseteq S\} = \bigcap_{b \in B} S_{-B}$
- Binary dilation transformation of *S* by *B*
  - the set of all those points *s* such that the translated *Bs* intersects *S*, and is equivalent to the union of all translates.
  - $S \oplus B = \{s: B_s \cap S \neq \varnothing\} = \bigcup_{b \in B} S_{-B}$
- The dilation with an elementary structuring template expands the set with a uniform layer of elements, while the erosion operator eliminates a layer from the set.
- Multiscale erosions and dilations are

  - $(S \ominus B) \ominus B \ominus \ldots \ominus B = (S \ominus nB)$,

  - $(S \oplus B) \oplus B \oplus \ldots \oplus B = (S \oplus nB)$,

  where $nB = B \oplus B \oplus \ldots \oplus B$ and n is the number of transformation cycles.

Mathematical Morphology (cont)

Binary MM (cont)

- By employing erosion and dilation of *S* by *B,* opening and closing transformations are further represented as:
  - $S \circ B = ((S \ominus B) \oplus B))$
  - $S \bullet B = ((S \oplus B) \ominus B))$
- After eroding *S* by *B*, the resultant eroded version is dilated to achieve the opened version of *S* by *B.*
- Similarly, closed version of *S* by *B* is obtained by first performing dilation on *S* by *B* and followed by erosion on the resultant dilated version.
- Multiscale opening and closing transformations are implemented by performing erosions and dilations recursively as shown below.

  - $(S \circ nB) = [(S \ominus nB) \oplus nB)]$,

  - $(S \bullet nB) = [(S \oplus nB) \ominus nB)]$,

  where *n* is the number of transformations cycles.

Mathematical Morphology (cont)

<u>Grey-scale MM</u>

- Grey-scale dilation and erosion operations - expansion and contractions respectively
- Let *f*(*x*,*y*) be a function on Z^2, and *B* be a fixed structuring element of size one. The erosion of DEM, *f*(*x*) by *B* replaces the value of *f* at a pixel (*x*, *y*) by the minima values of the image in the window defined by the structuring template *B*
  - $(f \ominus B)(x,y) = \min_{(i,j)\in B}\{f(x+i, y+j)\}$,
- The dilation of DEM, *f*(*x*) by *B* replaces the value of *f* at a pixel (*x*, *y*) by the maxima values of the image in the window defined by the structuring template *B*
  - $(f \oplus B)(x,y) = \max_{(i,j)\in B}\{f(x-i, y-j)\}$,
- In other words, ($f \ominus B$) and ($f \oplus B$) can be obtained by computing *minima* and *maxima* over a moving template *B*, respectively.
- Erosion is the dual of dilation :
  - Eroding foreground pixels is equivalent to dilating the background pixels.

Mathematical Morphology (cont)

<u>Grey-scale MM (cont)</u>

- Opening and closing are both based on the dilation and erosion transformations.
- Opening of DEM, *f* by *B* is achieved by eroding *f* and followed by dilating with respect to *B*, $(f \circ B) = [(f \ominus B) \oplus B]$,
- Closing of *f* by *B* is defined as the dilation of *f* by *B* followed by erosion with respect to B, $(f \bullet B) = [(f \oplus B) \ominus B]$,
- Opening eliminates specific image details smaller than *B*, removes noise and smoothens the boundaries from the inside, whereas closing fills holes in objects, connects close objects or small breaks and smoothens the boundaries from the outside.
- Multiscale opening and closing can be performed by increasing the size (scale) of the structuring template *Bn*, where *n* = 0, 1, 2,…, N. These multiscale opening and closing of *f* by *B* are mathematically represented as:

  $(f \circ B_n) = \{[(f \ominus B) \ominus B \ominus \ldots \ominus B] \oplus B \oplus B \oplus \ldots \oplus B\} = [(f \ominus nB) \oplus nB]$,

  $(f \bullet B_n) = \{[(f \oplus B) \oplus B \oplus \ldots \oplus B] \ominus B \ominus B \ominus \ldots \ominus B\} = [(f \oplus nB) \ominus nB]$,

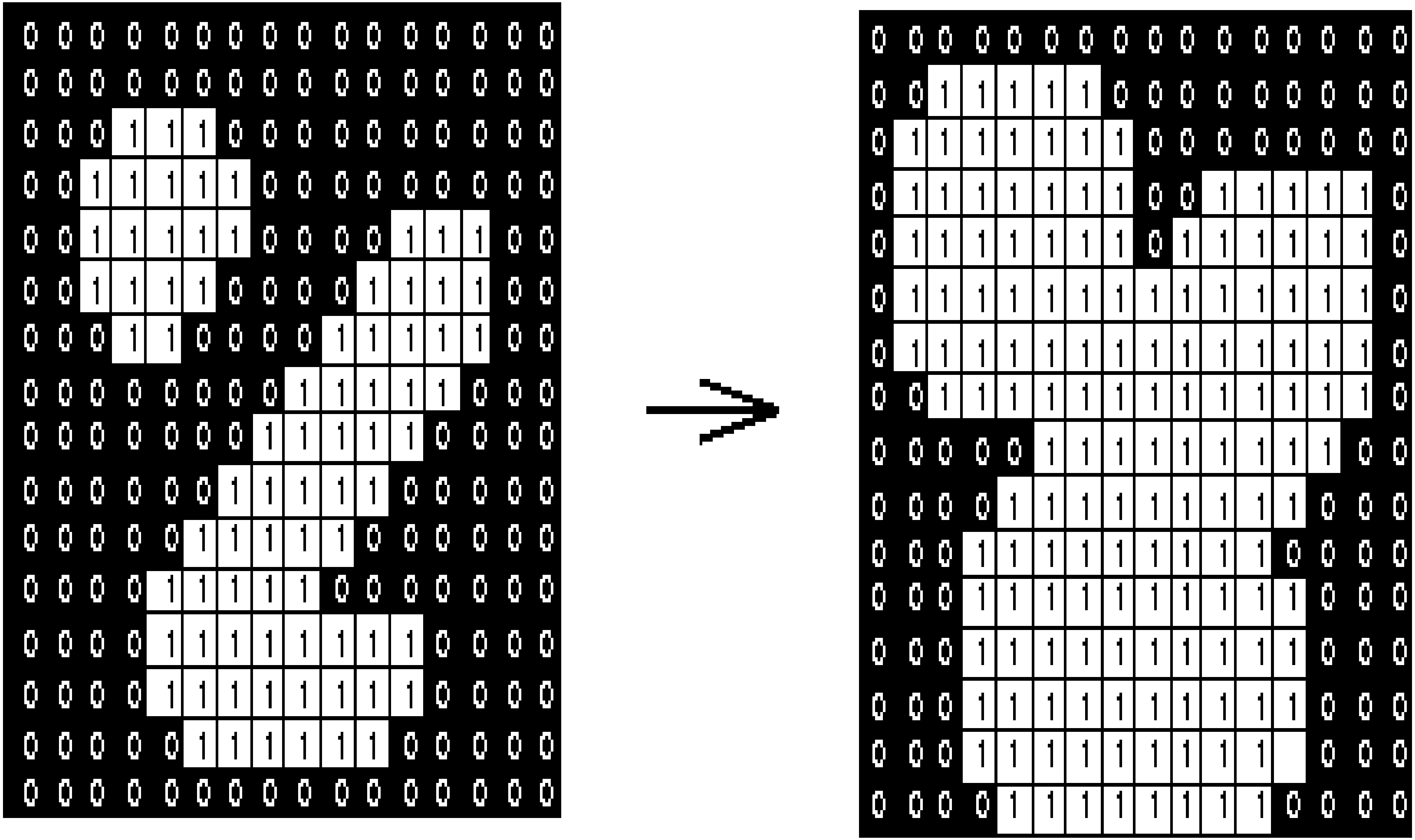
  at scale n = 0, 1, 2,…, N.
- Performing opening and closing iteratively by increasing the size of *B* transforms the DEM into lower resolutions correspondingly.

Mathematical Morphology (cont)

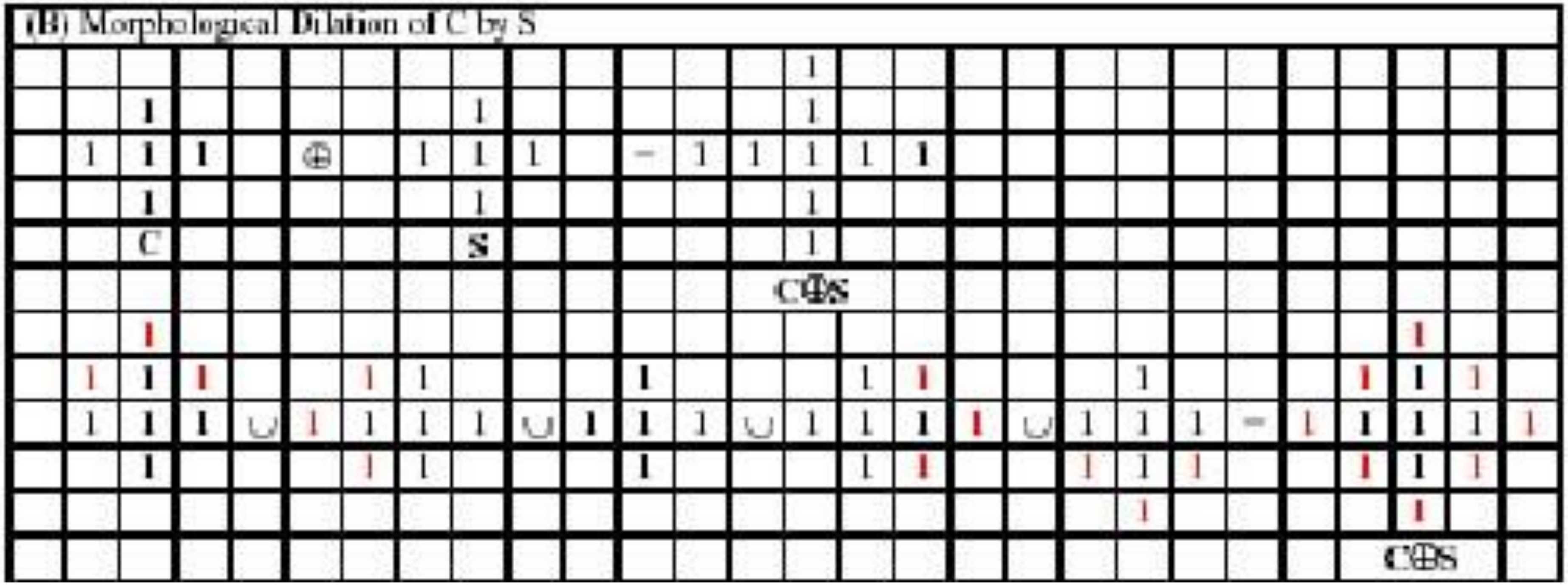
- Multiscale opening and closing of DEM by *nB* effect spatially distributed elevation regions in the form of smoothing of contours to various degrees. The shape and size of *B* control the shape of smoothing and the scale respectively.
- Important problems like feature detection and characterisation often require analysing DEMs at multiple spatial resolutions. Recently, non-linear filters have been used to obtain images at multi-resolution due to their robustness in preserving the fine details.
- Advantages of mathematical morphology transformations
  - popular in object recognition and representation studies.
  - The non-linearity property in preserving the fine details.

- ♦ Granulometries (via multiscale Opening)
- ♦ Anti-Granulometries (via multiscale Closing)

Effect of Dilation using 3X3 structuring element



Steps in Dilation of C by S



Effect of Erosion using 3X3 structuring element

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0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Steps in Erosion of C by S

(A) Morphological Erosion of C by S

	1						1							0													
1	1	1		⊖		1	1	1		=			0	1	0												
	1						1							0													
	C						S							C⊖S													
	0				1				1				1				1				0						
1	1	1	∩	0	1	1	∩	1	1	1	∩	1	1	0	∩	1	1	1	=	0	1	0					
	1				1				1				1				0				0						
																					C⊕S						

Effect of Opening using 3X3 structuring element

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Steps in Opening of C by S

(C) Morphological Opening of C by S

1	1	1			1			0	0	0			1				1	
1	1	1	$\ominus$	1	1	1	=	0	1	0	$\oplus$	1	1	1	=	1	1	1
1	1	1			1			0	0	0			1				1	
	C				S				$C \ominus S$				S				$C \ominus S \oplus S$	

Effect of Closing using 3X3 structuring element

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

→

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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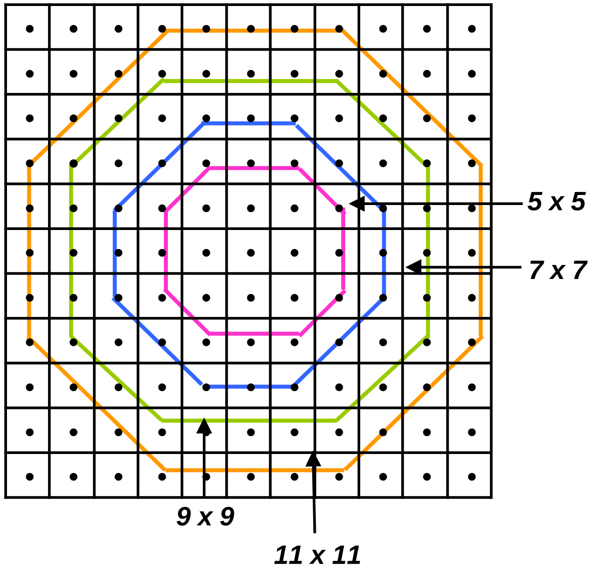
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Steps in Closing of C by S

(D) Morphological Closing of C by S

											1	1	1														
1	1	1				1				1	1	1	1	1					1				1	1	1		
1	0	1	⊕		1	1	1		=	1	1	1	1	1		⊖		1	1	1	=		1	1	1		
1	1	1				1				1	1	1	1	1					1				1	1	1		
	C					S					1	1	1						S					C			
												C⊕S												C⊕S⊖S			



Octagonal symmetric structuring elements of various primitive sizes ranging from 5 × 5 to 11 × 11. These primitive sizes can be considered as B.

Multiscale Opening and Closing

$$M \circ nB = \{[(M \ominus B) \ominus B \ldots \ominus B] \oplus B \oplus B \ldots \oplus B\} = [(M \ominus nB) \oplus nB]$$

$$M \bullet nB = \{[(M \oplus B) \oplus B \ldots \oplus B] \ominus B \ominus B \ldots \ominus B\} = [(M \oplus nB) \ominus nB]$$

- Multiscale grayscale transformations (erosion, dilation, opening, and closing), at scale $n = 0,1,2,\ldots,N$, are defined as follows:

$$(f \ominus nB) = (f \ominus B) \ominus B \ominus B \ominus \ldots \ominus B$$

$$(f \oplus nB) = (f \oplus B) \oplus B \oplus B \oplus \ldots \oplus B$$

$$(f \circ nB) = [(f \ominus nB) \oplus nB]$$

$$(f \bullet nB) = [(f \oplus nB) \ominus nB]$$

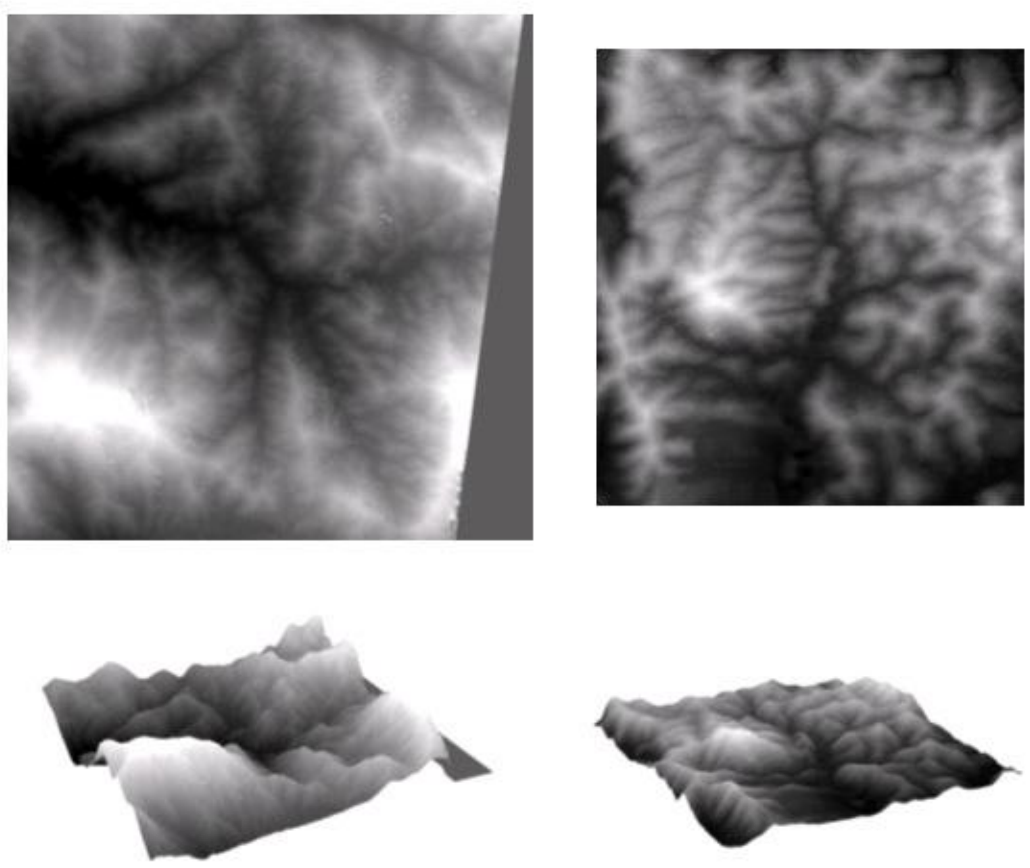
Study area specification

- ♦ SPOT X-Band data of Machap Baru reservoir situated in Melaka state, Malaysia with spatial resolution of 20 m acquired on 28/2/1998 situated in between 2^o 15’ - 2^o25’ N. Latitude and 102^o 15’ - 102^o 23’ E.Longitude.

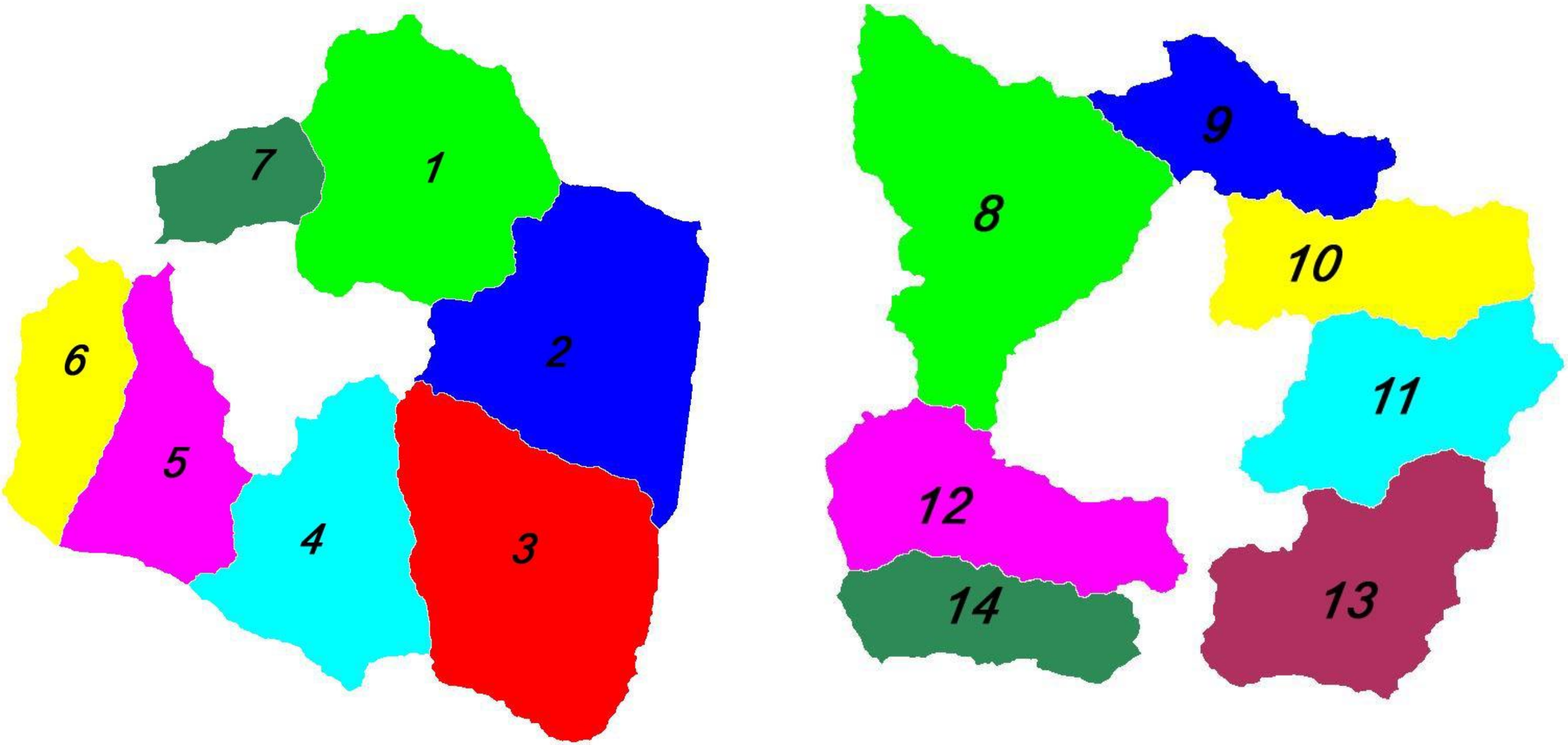
- ♦ Surveyed topographic map of scale 1:50000 for the region Machap Baru and Gunung Ledang.

- ♦ Data collected from Department of Irrigation and Drainage.

TOPSAR DEM

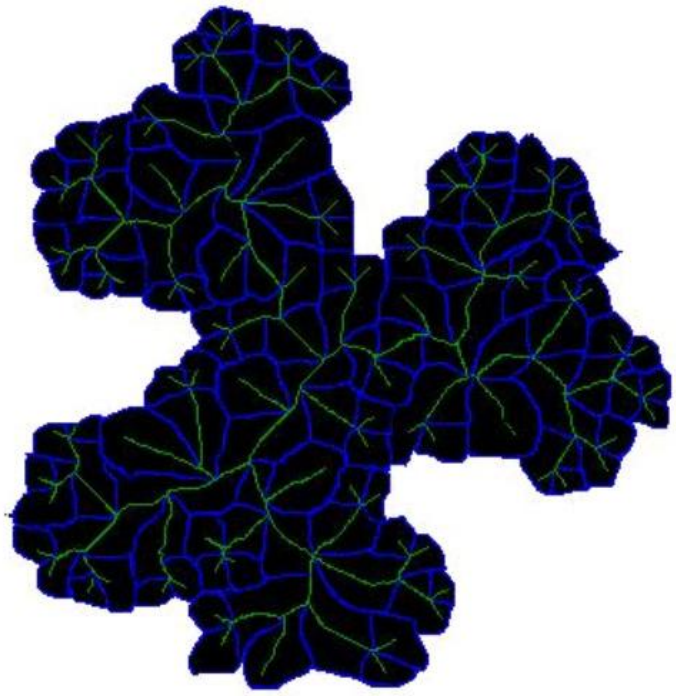


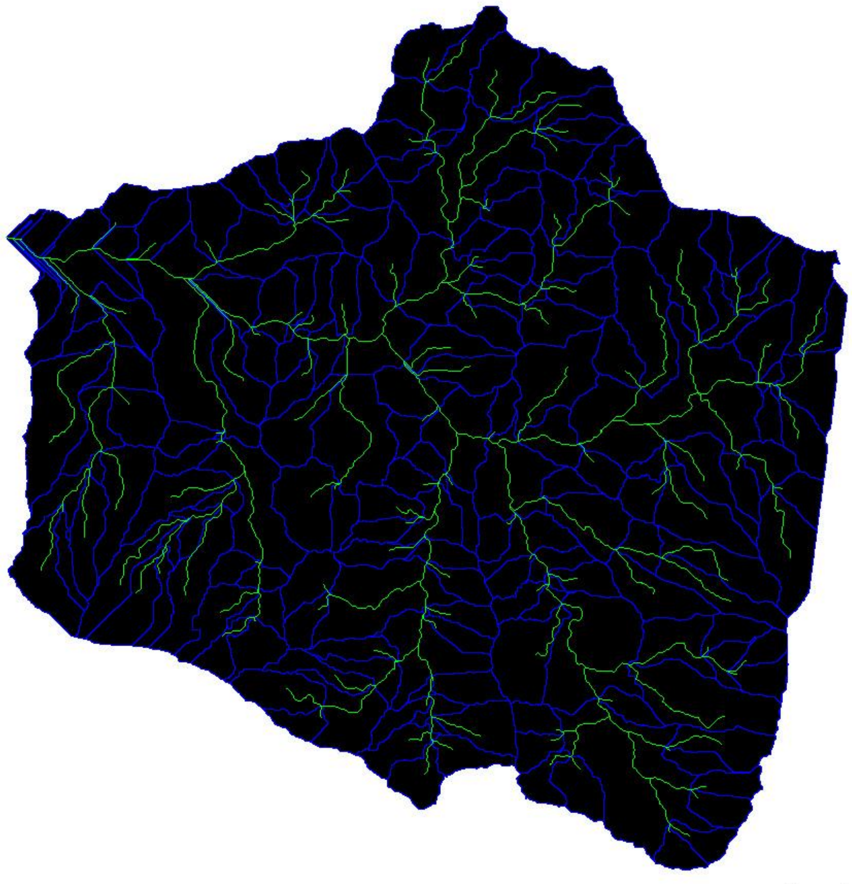
Networks extraction and their properties : Sub-basins delineation



The example of a few sub-basins delineated from Cameron Highlands and Petaling DEM is illustrated in figures above.

Decomposed basins and networks



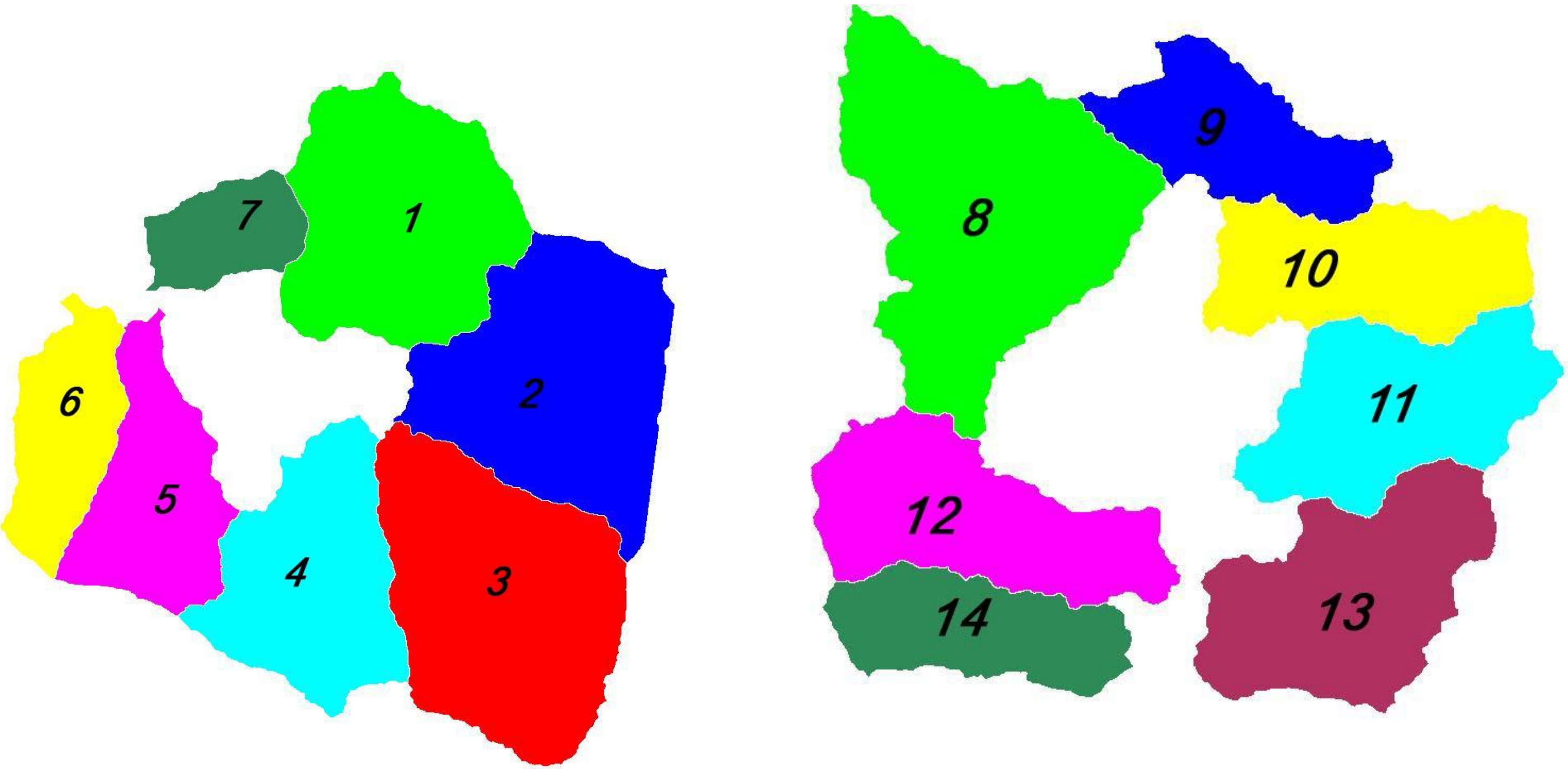


Fractal and multiscale analyses of planar geophysical networks

Fractal and multiscale analyses of networks

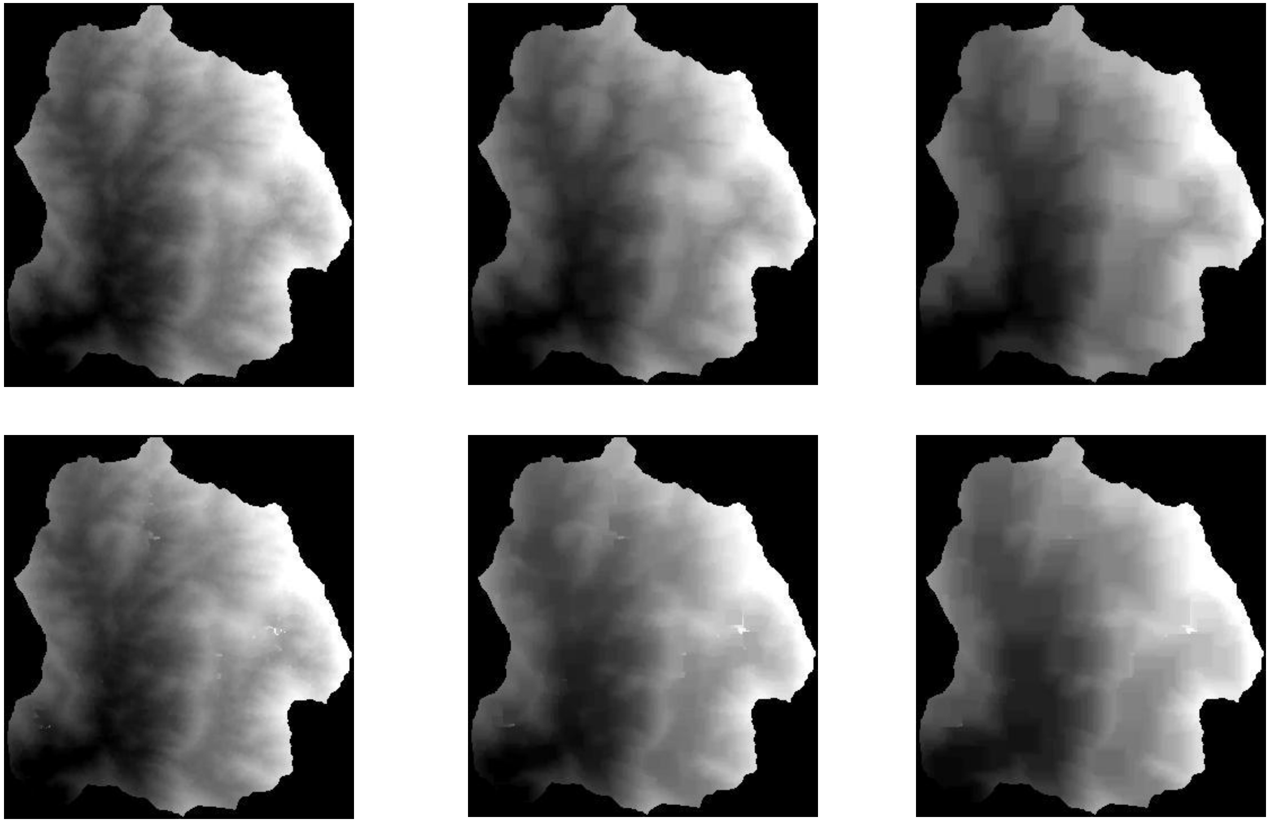
- A new approach of fractal dimension estimation is suggested in this analysis.
- Firstly, multiscale DEMs are generated *via* multiscale opening and closing transformations. The resolution of DEM becomes coarser with increasing cycle of opening/closing transformation.
- Both ridge and channel networks are extracted from these multiscale DEMs.
- A scaling exponent is derived by plotting the length of the network as function of the radius of structuring element employed to generate multiscale DEMs.
- This relationship possesses a linear property on a log-log graph. The exponent value derived from the best fit line is fractal-like scaling exponent.
- The derived fractal dimension is resolution-independent as the networks are extracted from basins of multiple resolutions. As compared to box-counting dimension, which describes the space-filling property of networks, the new proposed fractal dimension complements the existing methods in terrain characterisation.

Fractal and multiscale analyses of networks



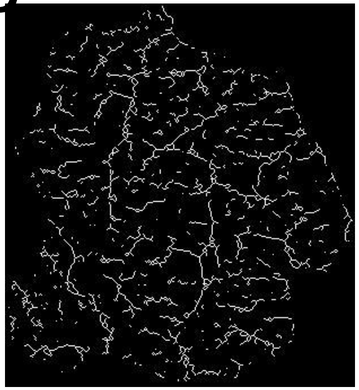
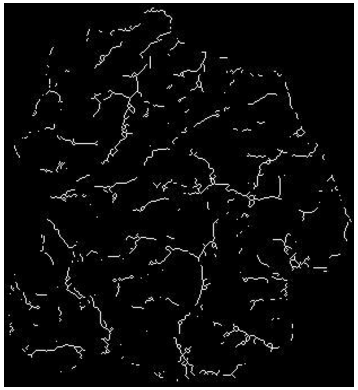
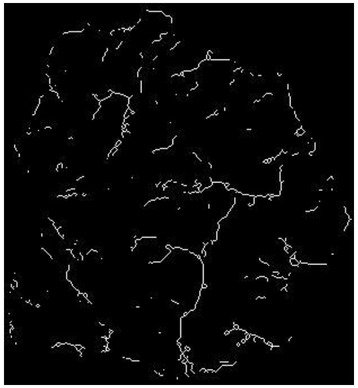
Fractal and multiscale analyses of networks

- Morphology Opening and Closing
- Multiscale DEM images (Opening and Closing by square structure element 3X3 to 21X21)

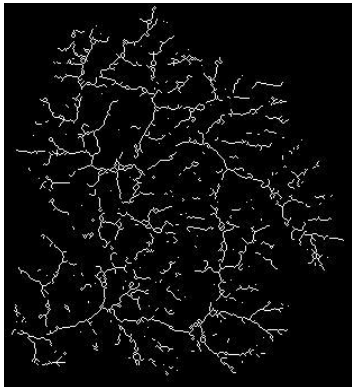
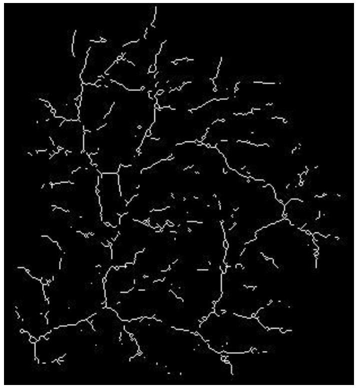
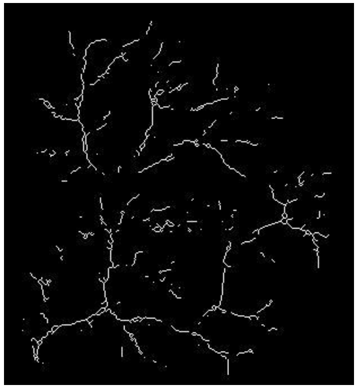


Fractal and multiscale analyses of networks : Post processing

- Thresholding
- Thinning
- Ridge

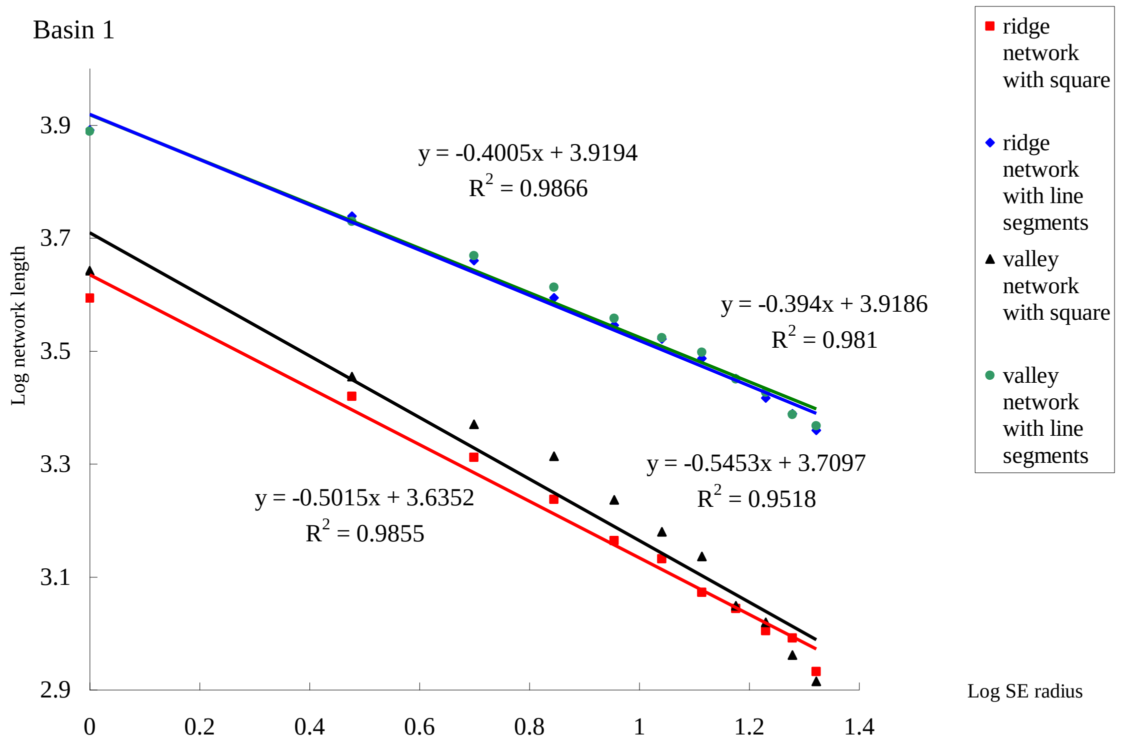




- Channel

Fractal and multiscale analyses of networks : Fractal Dimension

- Graph: Network length vs scale



Fractal and multiscale analyses of networks : Fractal Dimension

	Network extraction using line segments		Network extraction using square template	
Basin	Fractal dimension derived from ridge network.	Fractal dimension derived from channel network.	Fractal dimension derived from ridge network.	Fractal dimension derived from channel network.
Basin 1	1.4005	1.394	1.5015	1.5453
Basin 2	1.4097	1.4596	1.5184	1.5941
Basin 3	1.4204	1.3883	1.585	1.5926
Basin 4	1.5057	1.4068	1.7092	1.5301
Basin 5	1.4262	1.3402	1.5729	1.6233
Basin 6	1.3656	1.3314	1.5403	1.5412
Basin 7	1.3927	1.3468	1.5271	1.4273
Basin 8	1.3505	1.3688	1.512	1.4442
Basin 9	1.3349	1.3358	1.4251	1.3989
Basin 10	1.3781	1.357	1.5183	1.437
Basin 11	1.3137	1.3072	1.4263	1.3982
Basin 12	1.341	1.3342	1.5228	1.4644
Basin 13	1.299	1.3075	1.452	1.3735
Basin 14	1.3842	1.3367	1.5083	1.4658

Fractal and multiscale analyses of networks : Fractal Dimension

- This relationship depicts that similar trends have been followed for both ridge and channel connectivity networks, which shows the duality of both networks. It also describes the scaling properties of the terrain, where the density of the networks decreases as the resolution decreases. This change is due to the fact that the diffuse character of the basin increases as the size of the structuring template increases. This relation can be reversed and estimation of lengths of these networks can be made from coarse scale information.
- The lengths of channel and ridge networks extracted by employing line segment structuring elements are significantly more than that of the networks extracted by convex type of square template.
- The gradients of best fit lines of these plots indicate that the rate of change in the lengths of the networks across multiple resolutions. The rate derived by combination of segment-like structuring elements is slower than that of the networks derived by square element.
- Intricacy of the networks observed for Cameron sub-basins is denser as compared to the intricacy of Petaling networks. In general, hilly terrain possesses higher value of exponent as compared to non-hilly terrain. The reason is the rate of change in the elevation of hilly terrain across resolutions is higher than non-hilly terrain. Relatively, the network intricacies will also change more rapidly for hilly terrain.
- Since the power-law exponent is sensitive to the shape of structuring elements (shape-dependent), its value would be related to the shape and roughness of the terrain. Thus, these power laws can be related to terrain roughness characteristics.
- The analyses of networks extracted by structuring elements of each direction provide new insight to understand the direction-specific terrain complexity.

Granulometric analysis of digital topography

Granulometric analysis

- Morphological multiscaling transformations are shown to be a potential tool in deriving meaningful terrain roughness indices. Resolution constraints is one of the limitations in DEM analyses. In order to overcome these limitations, granulometric approach (a branch of mathematical morphology) is a potential approach because it provides scale-independent surficial roughness indices.
- Consider two different basins of two different physiographic setups (Cameron and Petaling regions) that possess similar topological quantities, their networks may be topologically similar to each other. But the processes involved therein may be highly contrasting due to their different physiographic origins. Under such circumstances, the results that exhibit similarities in terms of topological quantities and scaling exponents would be insufficient to make an appropriate relationship with involved processes.
- Therefore, granulometric approach is proposed to derive shape-size complexity measures of basins. This approach is based on probability distribution functions computed for both protrusions and intrusions (in other words *supremums* and *infimums*) of various degrees of sub-basins.
- This granulometry-based technique is tested on sub-basins with various sizes and shapes decomposed from DEM's of two distinct geomorphic regions, i.e. Cameron Highlands and Petaling region of Peninsular Malaysia.

Granulometric analysis

- Multiscale opening till completely black
- Multiscale closing till completely white
- Subtraction

$$PS_f(+n,B) = A[(f \circ B_n) - (f \circ B_{n+1})], 0 \le n \le N$$

$$PS_f(-n,B) = A[(f \bullet B_n) - (f \bullet B_{n-1})], 1 \le n \le K$$

- Probability function

$$ps(n,f) = \frac{A(f \circ B_n) - A(f \circ B_{n+1})}{A(f \circ B_0)}, n = 0,1,2,\ldots,N$$

$$ps(-n,f) = \frac{A(f \bullet B_n) - A(f \bullet B_{n-1})}{A(f \bullet B_K)}, n = 1,2,\ldots,K$$

- Average size

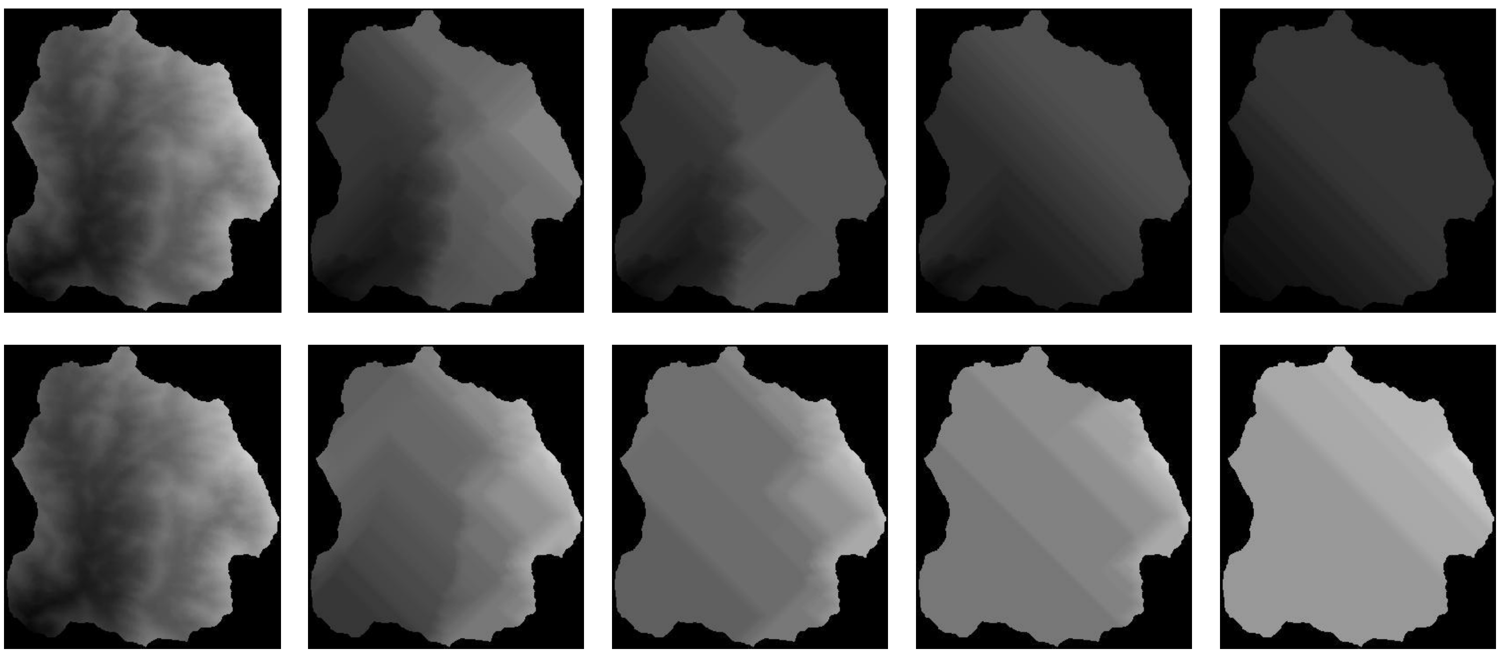
$$AS(f/B) = \sum_{n=0}^{N} n\,ps(n,f)$$

- Average roughness

$$H(f/B) = -\sum_{k=0}^{n} ps(n,f) \log ps(n,f)$$

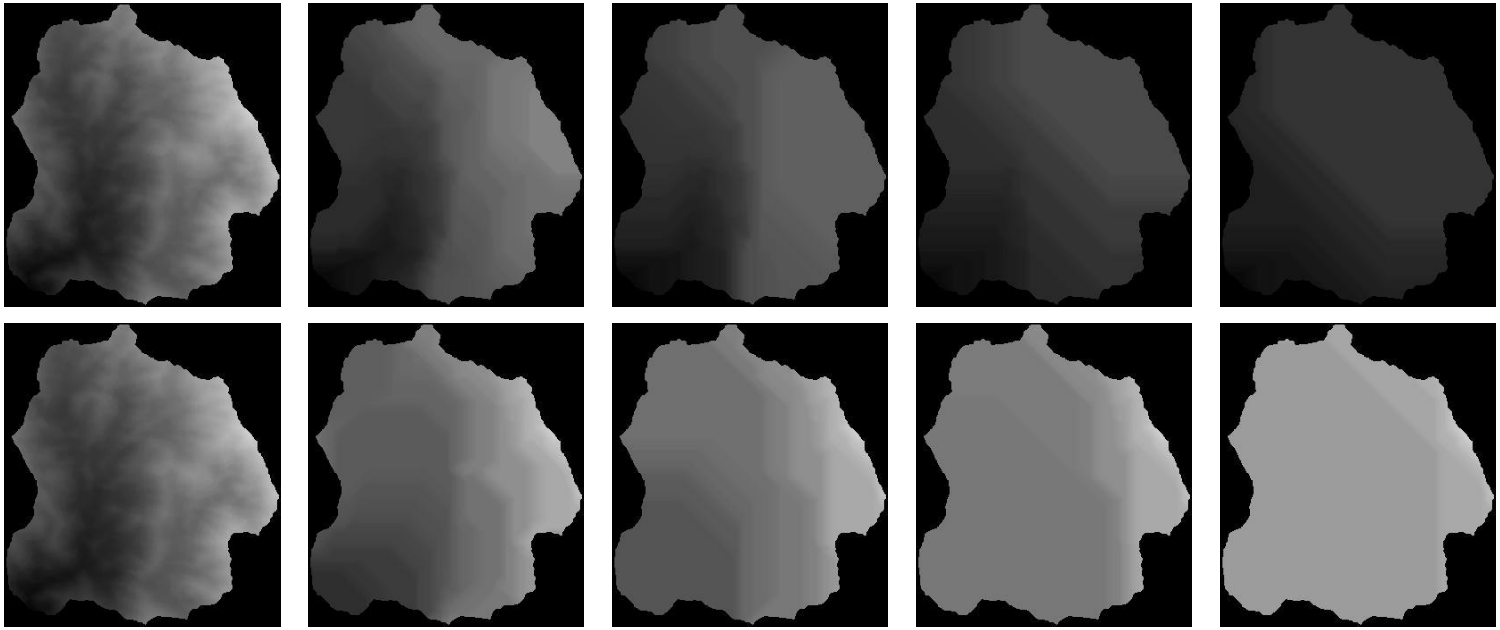
Granulometric analysis :
Multiscale opening/closing by rhombus

- Scale 1 , 40, 80, 120, 160



Granulometric analysis :
Multiscale opening/closing by octagon

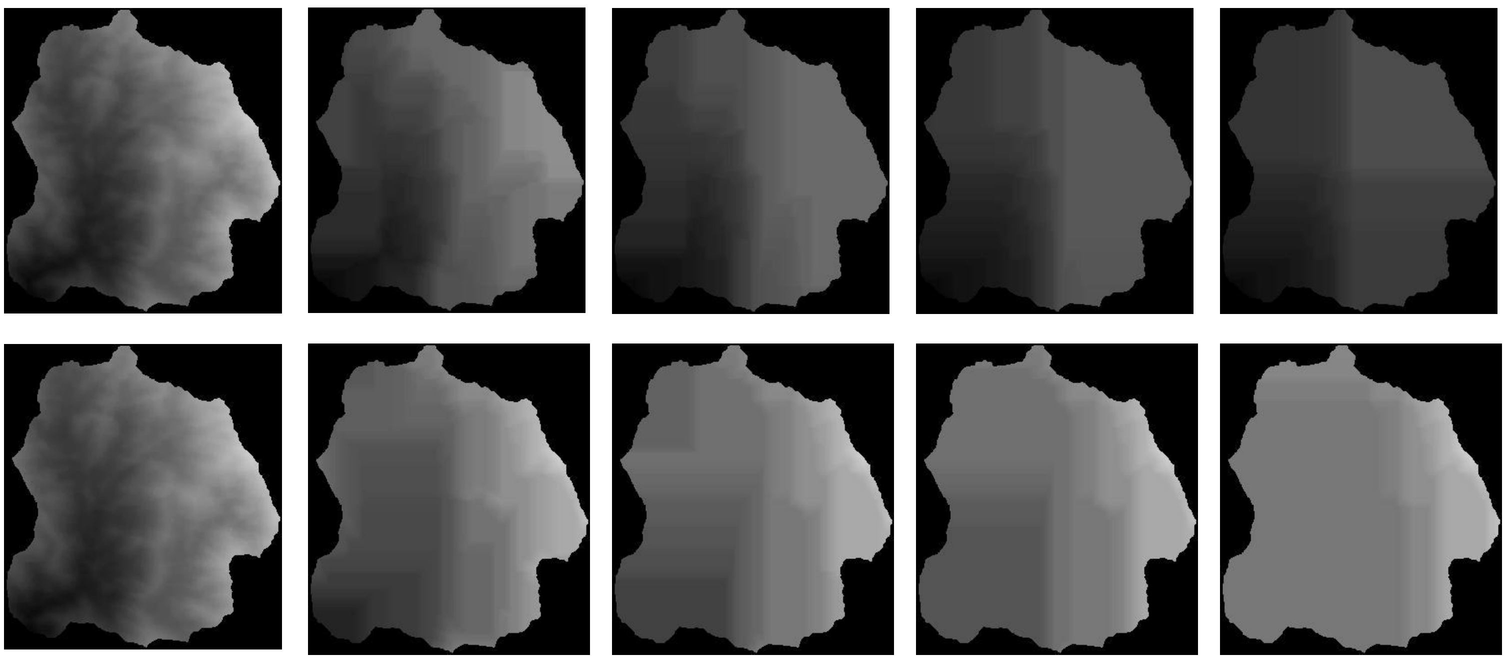
- Scale 1 , 30, 60, 90, 120



Granulometric analysis :

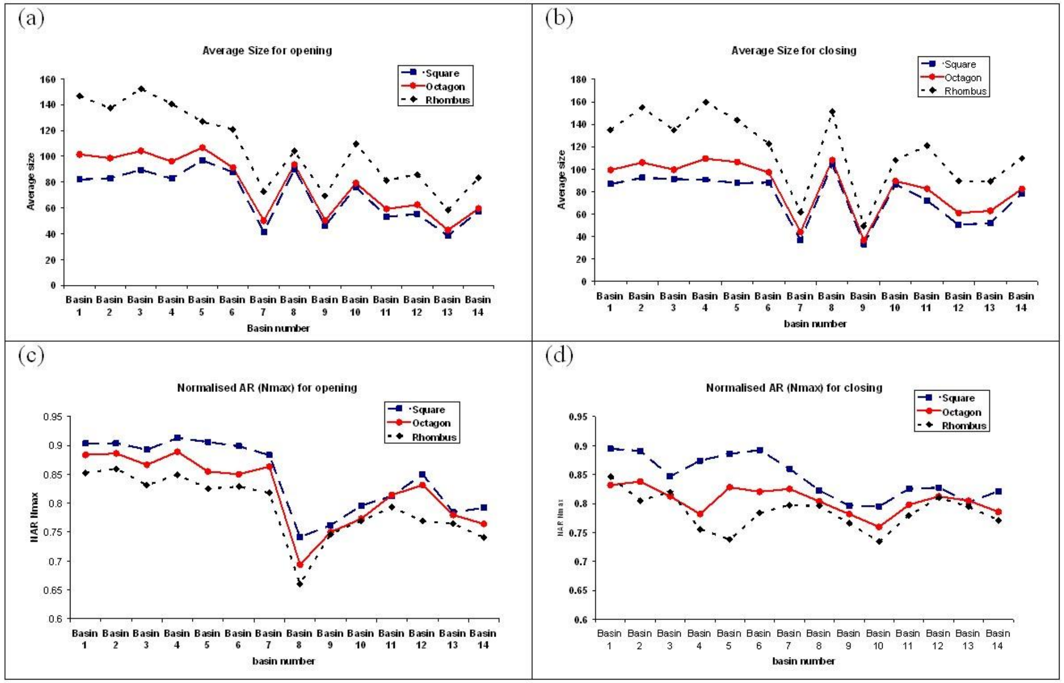
Multiscale opening/closing by square

- Scale 1 , 20, 40, 60, 80



Granulometric analysis : Basin wise analysis

- Average size – 14 sub-basins
- Average roughness – 14 sub-basins



Granulometric analysis : Basin wise analysis

- The number of iterations required to make each sub-basin either become darker or brighter depends on the size, shape, origin, orientation of considered primitive template used to perform multiscale openings or closings, and also on the size of the basin and its physiographic composition. More opening/closing cycles are needed when structuring element rhombus is used, and it is followed by octagon and square.
- Mean roughness indicates the shape-content of the basins. If the shape of SE is geometrically similar to basin regions, the average roughness result possesses lower analytical values. If the topography of basin is very different from the shape of SE, high roughness results are produced, which indicate that the basin is rough relative to that SE. In general, all basins are rougher relative to square shape as highest roughness indices are derived when square is used as SE.
- A clear distinction is obvious between the Cameron and Petaling basins. Generally, roughness values of Cameron basins are significantly higher than that of Petaling basins.
- The terrain complexity measures derived granulometrically are scale-independent, but strictly shape-dependent. The shape dependent complexity measures are sensitive to record the variations in basin shape, topology, and geometric organisation of hillslopes.
- Granulometric analysis of basin-wise DEM's is a helpful tool for defining roughness parameters and other morphological/topological quantities.