Due Date : 2 June, 2016

1) There are 21 students in a class. Out of them, 14 students like pizza and 10 like donuts. How many students like both pizza and donuts ?

2) (i) How many numbers from 1 to 30 are not divisible by 3 or 5 ? (Hint : If A is the set of numbers from 1 to 30 divisible by 3, and B is the set of numbers from 1 to 30 divisible by 5, we are looking for elements that do not belong to $A \cup B$)

   (ii) How many numbers from 1 to 50 are not divisible by 2 or 4 or 7 ?

   (iii) Manually do (i) and (ii) by listing out the numbers and crossing out any time you see a number divisible by $3, 5$ in (i), and $2, 4, 7$ in (ii). Compare the answers to what you obtained, presumably using inclusion-exclusion principle.

3) We saw Cantor's diagonal argument in class to prove that the set of real numbers has bigger cardinality than the set of natural numbers. Let m denote the number we are trying to construct using the diagonal argument. In Cantor's argument, is it possible to consider pairs of digits rather than single digits, that is, suppose we look at the first two digits of the first real number on our list, and if they are 22, then we let the first two digits of m be 22. If the first two digits are 22, then we make the first two digits of m be 44. Similarly, the next two digits of the next real number in our list determine the next two digits of m and so on. If this procedure would still produce a number not our list, please provide the details for such a method. If the procedure does not work, explain or illustrate why it does not.

4) In most proofs using induction, first we look at patterns and make a guess as to what the answer could be in the general case. But as we have seen before inductive reasoning may lead us to wrong conclusions. The principle of mathematical induction helps us in using the inductive way of thinking to prove results. Note that if the conclusion is incorrect, induction or for that matter, any kind of argument cannot prove it.
   Compute the sum of the first few consecutive odd numbers. $1, 1+3, 1+3+5, 1+3+5+7, \cdots$. Looking at the pattern, make a guess for the formula for the sum of the first n odd numbers. Can you prove it using the principle of mathematical induction ?

5) In class, we defined the factorial of a natural number in the following manner : $n! = n \times (n-1) \times \cdots \times 2 \times 1$. We also made a comment that the factorial of n grows faster than any exponential function. In this exercise, we try to prove a version of that. Using the principle of mathematical induction, prove that $n! > 2^n$ for $n \geq 4$.
   N.B.: Here the base case of the induction is not 1 but 4.