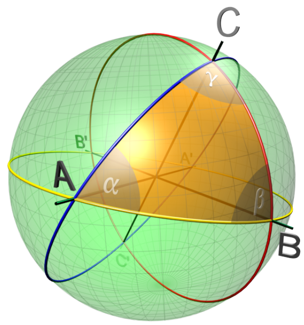
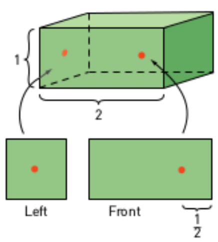
Due date : July 23, 2016

1) A spherical triangle is a geometrical figure on the sphere which has three sides each of which is part of a great circle. Draw three great circles on the sphere. How many spherical triangles have you made?



2) How many regions is the sphere divided into by 5 great circles?

3) For each pair of points on the boxes, describe the shortest path from one point to another, for a bug walking on the walls.



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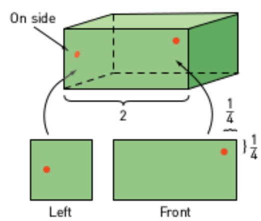

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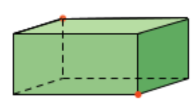

4) In a regular octahedron, a common vertex is shared by four faces each of which is an equilateral triangle. What is the shortest distance between points in opposite faces amongst these four faces, for a bug walking on the walls? For ease of computation, solve the problem in the following two cases atleast (for full credit).

(a) centroids of the opposite faces,

(b) 'diametrically' opposite vertices.

(For the general computation represent the points by three non-negative numbers which sum to 1, weights on the position vectors of the vertices. $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ corresponds to centroid. $(1, 0, 0), (0, 1, 0), (0, 0, 1)$ represent the vertices. Here the order of the vertices would matter, so keep track of that.)

5) Take a cube and put a point in the middle of each face. Draw straight lines to the middles of each of the sides of the face, thus producing a plus + sign on each face. The kinked line that goes from the center of one face to the center of an adjacent face forms a bent edge on this cubical world. Thus we have created eight bent triangles whose vertices are the centers of the faces of the cube. Compute the sum of angles of each of these triangles. What is the sum

of all the angles of all the triangles?

Put a vertex at the center of each face of a regular tetrahedron and connect adjacent faces over the center of each edge. Compute the sum of angles of each of the "bent" triangles similar to above. What is the sum of all the angles of all the triangles?

Put a vertex at the center of each face of a regular dodecahedron and connect adjacent faces over the center of each edge. Compute the sum of angles of each of the "bent" triangles. What is the sum of all the angles of all the triangles?

For each case, compute the difference between sum of all angles of all "bent" triangles and $180° \times n$ where n is the number of triangles.
(On the Euclidean plane, the sum of angles of a triangle is 180°. In this exercise, we illustrate the fact that on a polyhedron the sum of angles of a triangle exceeds 180°. The difference tells us by how much in total, and if computed correctly will turn out to be a number independent of the kind of polyhedron considered. In fact, the difference is related to the Euler characteristic of the shape that is being triangulated. The same computation for a triangulation (or polygonization) of a torus will give a different answer. The cube, tetrahedron, dodecahedron are essentially polygonizations of the sphere.)