Due date : June 23, 2016

1) In class, we saw examples of divisibility tests for quickly deciding whether a given number is divisible by $3, 9$ or 11. In this problem, we investigate a divisibility test for 7.

   (i) Remove the last digit and double it,

   (ii) Subtract it from the truncated original number

   (iii) Continue doing this until only one digit remains.

   (iv) If this is 0 or 7, then the original number is divisible by 7.

   For example, the steps given above for the number 97804 would be the following. $97804 \to 9780 - 2 \times 4 = 9772 \to 977 - 2 \times 2 = 973 \to 97 - 2 \times 3 = 91 \to 9 - 2 \times 1 = 7$. Thus, our divisibility test tells us that 97804 is divisible by 7 and indeed, $97804 = 7 \times 13972$.
   Why does this test work ? Formulate a way of showing that, if the original number is divisible by 7, the transformed numbers are too and vice versa.

2. A task takes a fixed number of days (≤ 7) to finish and I do not work on weekends (Sat, Sun). I start working on a Thursday and finish four tasks one after the other, and tell you that I finished them on a Monday. Can you deduce how many days it takes to finish a task?

3. Find an integer x such that $13x \equiv 15 \pmod{29}$ i.e. $13x$ leaves a remainder of 15 when divided by 29. (Hint : Use Euclid's algorithm to compute inverse of 13 modulo 29 and multiply it to 15. But you must provide a brief explanation why this should work.)

4. In this exercise, we would like to find remainders for the power of a number, modulo 11. For each of $2, 3, 4$, compute the remainders of $2^n, 3^n, 4^n$ ($1 \leq n \leq 22$) when divided by 11. Write the remainders sequentially, in three separate rows corresponding to $2, 3$ and 4. Do the numbers appearing in the sequence have some pattern? Identify it and describe it.

5. (An encoding scheme) To each letter in the alphabet I assign a number corresponding to its position ($A \to 1, B \to 2, \cdots, Z \to 26$ and space corresponds to 0. For the moment, assume that the message has no punctuation.) Given a message, we use the following encoding scheme.

   (i) Write the corresponding numbers for each letter (and space).

   (ii) Multiply 11 to each of the numbers.

   (iii) Compute the remainders for each of the numbers when divided by 27.

   (iv) Translate the sequence of numbers back to the letters (and space).

   For instance, MATH IS FUN = 13 1 20 8 0 9 19 0 6 21 14 $\to$(multiply 11) 143 11 220 88 0 99 209 0 66 231 154 $\to$(compute remainder) 8 11 4 7 0 18 20 0 12 15 19 $\to$ HKDG RT LOS.
   Now, decode the following message : FCIIAFD KSTJAI.