

Due date : June 16, 2016

- 1) (Erasthones' sieve) List numbers from 2 to 50. Firstly, cross out the even numbers from the list and label them with 2 on top. Then we take the next number, say m , which has not been crossed out from the list and cross out all its multiples from the list and label them with m . If a multiple has previously been crossed out and labelled, we just ignore it. We continue this procedure until we run out of numbers. List the numbers in the list which have not been crossed out. What do the labels on top of the crossed out numbers signify ?
2. Write the following numbers as the sum of squares of four numbers in $\mathbb{N} \cup \{0\}$.

15, 23, 54, 78, 93

3. Find the two numbers for each part of this question. The greatest common divisor (GCD), least common multiple(LCM), and the sum of x, y are given.
 - (i) $\text{GCD}(x, y) = 12, \text{LCM}(x, y) = 72, x + y = 60$
 - (ii) $\text{GCD}(x, y) = 5, \text{LCM}(x, y) = 175, x + y = 60$
 - (i) $\text{GCD}(x, y) = 1, \text{LCM}(x, y) = 15, x + y = 16$
 - (i) $\text{GCD}(x, y) = 3, \text{LCM}(x, y) = 168, x + y = 45$
4. We can find inverses of natural numbers modulo a prime number p (for those numbers not divisible by p). We take $p = 13$ in this exercise. Compute the inverses of 2, 3, 5, 6, 7 modulo 13.
5.
 - (i) Find the last digit of 7^{25} .
 - (ii) Find the last two digits of 2^{100} .
 - (iii) Find the last two digits of 2^{100} in base 9.