

Due date : June 9, 2016

- 1) The local Indian take-out has a lunch special where one chooses one item from each of the following groups :

Biryani (mixed rice dish)	Bread	Curry	Dessert
Vegetarian Chicken Lamb	Naan Kulcha Parantha Chapati	Lentils Eggplant Rajma (Kidney beans) Aloo Mutter (Potatoes, Peas) Dhingri (Mushroom)	Gulab Jamun Rasgulla

- How many different lunch specials are possible ?
 - If one prefers to cancel the dessert and forego meat from the menu, then how many possibilities are there for the lunch special ?
- 2) In a class, a quiz consisting of multiple choice questions has been scheduled. The instructor wants to give the same questions to all students but in different orders. If there are 34 students enrolled in the class, what is the minimum number of questions needed for the quiz ?
- 3) We say that a coin is *fair* if there is no bias towards heads or tails when we toss it. We have seen in class that the number of possible outcomes for a sequence of n tosses is 2^n . For example, if we toss three times ($n = 3$), then the possibilities are $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. Note that in this case, the ways exactly two heads occur in our coin tosses are HHT, HTH, THH . But there is only one way of having three heads *i.e.* HHH . In a sequence of n tosses, the number of ways k heads can occur is given by $\binom{n}{k}$.
- For 10 consecutive coin tosses, compute how many ways 5 heads can occur. Then compute the number of ways 5 heads can occur in a row. Clearly the second condition is more restrictive. But how restrictive ? To get an intuition for that, compute the ratio of the number of ways in each instance.
 - In Pascal's triangle, what is the position of the biggest number in each row ? Can you reason why that should be the case.
 - I toss a coin 6 times. If there are 3 or 4 heads in the sequence, you pay me \$ 1.00. Otherwise I pay you \$ 1.00. Is this a fair game ?
- 4) In the following problems, identify the *pigeons* and the *pigeonholes* before applying the pigeonhole principle to solve the problem.
- There are several pairs of red and black socks all mixed up in a drawer. The room is dark. How many socks should I get out of the drawer to ensure that I have a pair of matching socks ?
 - Suppose a bag of Skittles contains four different colors (purple, yellow, green, orange). If there are 21 Skittles in the bag, show that there must be at least 6 Skittles of the same color.

- c. Penn has 10000 undergraduate students. Show that there is a group of 28 students with the same birthday.
- d. Five points are placed inside a unit square. Show that there must be two points no more than $\frac{1}{\sqrt{2}}$ units apart.
5. For the graphs below, do the following. Label the vertices. Note the degree of each vertex. Identify which of these graphs have a Eulerian circuit or a Eulerian path or neither. In case there is a Eulerian path or a Eulerian circuit trace it with arrows marking the direction of traversal.

