

Homework 4

Due on 1 August, 2013

You may solve any 4 of the 5 questions below correctly for full credit. But you are strongly encouraged to attempt all problems.

1

Find the following limits (if it exists) with justification :

$$\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}, \quad \lim_{(x,y) \rightarrow (0,0)} \frac{e^y \sin x}{x}$$
$$\lim_{(x,y) \rightarrow (1,1)} \frac{xy - y - 2x + 2}{x - 1}, \quad \lim_{(x,y) \rightarrow (2,2)} \frac{x + y - 4}{\sqrt{x + y} - 2}$$

2

At what points are the following functions continuous ?

i) $f(x, y, z) = \ln xyz$

ii) $f(x, y, z) = \frac{1}{|xy| + |z|}$

3

Find all the first-order and second-order partial derivatives of the following functions :

i) $f(x, y) = x^2 \tan(xy)$

i) $f(x, y) = \ln(x + y)$

4

Find the value of $\frac{\partial z}{\partial x}$ at the point $(1, 1, 1)$ if the equation

$$xy + z^3x - 2yz = 0$$

defines z as a function of the two independent variables x and y and the partial derivative exists.

5

Find the normal to the curve given by $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.