Homework 3

Due on 25 July, 2013

You may solve any 4 out of the 5 questions below correctly to get full credit. But you are strongly encouraged to attempt all problems.

1

Assume that you can shoot an arrow at a speed of 50 m/s. A helium-filled balloon is released from the ground 150 m away and rises vertically at a uniform speed of 5 m/s. How should you shoot the arrow right after the balloon is released so that you may deflate it ?

2

Find the length of the curve

$$\vec{r}(t) = \sqrt{2}t\hat{i} + \sqrt{2}t\hat{j} + (1 - t^2)\hat{k}$$

from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

3

Show that the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0,$$

has its largest curvature on its major axis and its smallest curvature on its minor axis.

4

Compute the TNB frame and the *curvature* and *torsion* for the space curve given by

$$\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t^2\hat{k}$$

5

Show that a planet in a circular orbit moves with a constant speed. (Hint : Use polar coordinates.)