Homework 1

Due on 11 July, 2013

Questions 1 and 2 are compulsory. Any two out of 3,4,5 submitted will be graded. But you are encouraged to attempt all the problems

1

$$\frac{dP}{dt} = 2P(P-3)(5-P)$$

Use a phase line analysis to sketch solution curves for P(t), selecting different starting values P(0). Which equilibria are stable, and which are unstable ?

$\mathbf{2}$

Consider a competitive-hunter model defined by

$$\frac{dx}{dt} = a\left(1 - \frac{x}{k_1}\right)x - bxy,$$
$$\frac{dy}{dt} = m\left(1 - \frac{y}{k_2}\right)y - nxy,$$

where x and y represent trout and bass populations, respectively. Perform a graphical analysis:

- i) Find the possible equilibirum levels.
- ii) Determine whether coexistence is possible.
- iii) Pick several starting points and sketch typical trajectories in the phase plane.
- iv) Interpret the outcomes predicted by your graphical analysis in terms of the constants a, b, m, n, k_1 , and k_2 .

3

The equation of an ellipse is given by

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$

- i) Find a parametric equation for the ellipse.
- ii) Find the point on the ellipse closest to (4, 5).

4

Find the area under one arch of the cycloid,

$$x = a(t - \sin t), y = a(1 - \cos t)$$

$\mathbf{5}$

Write inequalities to describe the following sets:

- i) The solid cube in the first octant bounded by the coordinate planes and the planes x = 2, y = 2, z = 2.
- ii) The half-space consisting of the points on and below the xy-plane.
- iii) The upper hemisphere of the sphere of radius 1 centered at the origin.
- iv) The exterior of the sphere of radius 1 centered at the origin.