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Remarks on the Type of Von Neumann Algebras of Local Observables in Quantum Field Theory*

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The von Neumann algebras of local observables associated with certain regions of space–time are believed to be factors. We show that these algebras are not of finite type. The commutant of the tensor product of two semifinite von Neumann algebras is analyzed with the aid of this result. The factors in question have the vacuum state as separating and cyclic vector. It is shown that a factor of type I_∞ with I_∞ commutant, and a subfactor of type I_∞ with I_∞ relative commutant have a common separating and cyclic vector. This settles negatively some conjectures aimed at proving that these factors are not of type I. An argument of Araki's showing that the factors associated with certain regions are not of type I is presented in simplified form.

I. INTRODUCTION

SOME attention has been given recently to the algebras of local observables associated with regions of space–time by a quantum field theory.[1–6] For certain regions, these von Neumann algebras are believed to be factors in the sense of Murray and von Neumann.[7] The question of the types[8] of the factors occurring is of some importance in this connection. Making use of the cyclicity and separating properties of the vacuum state for these factors, we show (Theorem 1) that they are of infinite type. This same result makes possible a direct proof (avoiding Hilbert algebras) of the known result[9] $(\mathcal{R}_1 \otimes \mathcal{R}_2)' = \mathcal{R}_1' \otimes \mathcal{R}_2'$ when $\mathcal{R}_1$ and $\mathcal{R}_2$ are semifinite von Neumann algebras (i.e., have no portion of type III). Section IV is devoted to this proof.

The strong separating and cyclicity properties of the vacuum state relative to the various factors seem to rule out their being of type I. The basic question is:

If $\mathcal{R}$, $\mathcal{R}'$, $\mathcal{R}_1$, and $\mathcal{R}_1' \cap \mathcal{R}$ are factors of type I_∞ and $\mathcal{R}_1 \subseteq \mathcal{R}$, can $\mathcal{R}$ and $\mathcal{R}_1$ have a joint generating and separating vector? (1.1)

In Sec. V we analyze cyclic and separating vectors for factors of type I_∞ with I_∞ commutants, and reduce some variants of (1.1)[10] to (1.1). In Sec. VI, we construct such a joint generating and cyclic vector (settling the associated conjectures[10] negatively).

The final section contains a simplified form of an argument of Araki's.[11] The uniqueness of the vacuum state as a translation invariant, together with the fact that it is separating, is used to show that the factor associated with a certain region of space–time is not of type I.

Question 1.1 arose in a conversation (October 1962) with A. S. Wightman (Theorem 1 was proved during this conversation).

II. NOTATION

As we have done in the introduction, we denote by $\mathcal{R}'$ the set of (bounded) operators commuting with all the operators of $\mathcal{R}$ ($\mathcal{R}'$ is called *the commutant* of $\mathcal{R}$). We use the symbol and terminology for an orthogonal projection operator interchangeably with the symbol and terminology for its range (the closed subspace on which it projects). If $\mathcal{R}$ is a family of operators and N a set of vectors, $[\mathcal{R}N]$ will denote the closed subspace spanned by vectors of the form Ax with A in $\mathcal{R}$ and x in N (so that, by the convention just adopted, $[\mathcal{R}N]$ will also denote the orthogonal projection operator on this subspace).

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1 H. Araki, J. Math. Phys. 4, 1343 (1963).
2 H. Araki and E. J. Woods, J. Math. Phys. 4, 637 (1963).
3 R. Haag, Proceedings of the Midwest Conference on Theoretical Physics, Minneapolis, Minnesota, 1961.
4 R. Haag and B. Schroer, J. Math. Phys. 3, 248 (1962).
5 I. E. Segal, *Mathematical Problems of Relativistic Physics* (American Mathematical Society, Providence, Rhode Island, 1963).
6 M. Guénin and B. Misra, "On the von Neumann algebra generated by the field operators" (mimeographed note, Institute of Theoretical Physics, Geneva).
7 F. J. Murray and J. von Neumann, Ann. Math. 37, 116 (1936).
8 See reference 7, especially pp. 171–172.
9 J. Dixmier, *Les algèbres d'opérateurs dans l'espace Hilbertien* (Gauthier-Villars, Paris, 1957), p. 102, Proposition 14.
10 See reference 6 (listed there as Conjectures B1 and B2).
11 See reference 1, especially Lemmas 10.1—10.3; and reference 2, especially Lemmas 4.1 and 4.2.

III. INFINITE TYPE

The von Neumann algebra of local observables associated with a bounded open region $\mathcal{O}$ of space–time by a quantum field is a factor which has the vacuum state ψ_0 as a separating and cyclic vector. If $\mathcal{O}_0$ is an open subregion of $\mathcal{O}$ with boundary at positive distance from the boundary of $\mathcal{O}$, its factor $\mathcal{R}_0$ is a *proper* subfactor of $\mathcal{R}$ (again with ψ_0 as separating and cylic vector). We prove

Theorem 1. The factor $\mathcal{R}$ is of infinite type.

This will be accomplished by establishing:

Lemma 2. If $\mathcal{R}_0$ is a proper sub von Neumann algebra of the von Neumann algebra $\mathcal{R}$, and x is a separating and cyclic vector for both $\mathcal{R}$ and $\mathcal{R}_0$, then $\mathcal{R}$ (and $\mathcal{R}_0$) are of infinite type.

Remark. Although the proof is somewhat simpler in the factor case, it seems worthwhile to establish this lemma for arbitrary von Neumann algebras. We shall do this.

Proof: We assume that $\mathcal{R}$ is finite and show that $\mathcal{R}_0 = \mathcal{R}$. Assuming $\mathcal{R}$ finite, $[\mathcal{R}x]$ ($=\mathcal{H}$, the underlying Hilbert space) is finite in $\mathcal{R}'$[12]; so that $\mathcal{R}'$ (and, similarly, $\mathcal{R}_0'$) are finite. Let D, D', D_0, and D_0' denote the center-valued dimension functions on $\mathcal{R}$, $\mathcal{R}'$, $\mathcal{R}_0$, and $\mathcal{R}_0'$, respectively,[13] each normalized so that the identity operator I has dimension I. Since $\mathcal{H} = [\mathcal{R}x] = [\mathcal{R}'x] = [\mathcal{R}_0x] = [\mathcal{R}_0'x]$, $D'([\mathcal{R}y]) = D([\mathcal{R}'y])$ and $D_0'([\mathcal{R}_0y]) = D_0([\mathcal{R}_0'y])$, for each y in $\mathcal{H}$, by virtue of the Coupling Theorem.[14] In particular, with P a central projection in $\mathcal{R}$, $[\mathcal{R}_0Px] = P = [\mathcal{R}'Px] \subseteq [\mathcal{R}_0'Px]$; so that $[\mathcal{R}_0'\mathcal{R}_0Px] \subseteq [\mathcal{R}_0'Px] \subseteq [\mathcal{R}_0'\mathcal{R}_0Px]$ and $D_0'([\mathcal{R}_0Px]) = D_0'(P) = D_0([\mathcal{R}_0'Px]) = [\mathcal{R}_0'Px]$ (since $[\mathcal{R}_0'Px]$ is $[\mathcal{R}_0'\mathcal{R}_0Px]$, a central projection in $\mathcal{R}_0$).[13] Now $[\mathcal{R}_0'Px]D_0'(P) = D_0'([\mathcal{R}_0'Px]P) = D_0'([\mathcal{R}_0'Px])$, so that $P \geq [\mathcal{R}_0'Px]$. Thus $P = [\mathcal{R}_0'Px] \in \mathcal{R}_0$; and the center of $\mathcal{R}$ is contained in that of $\mathcal{R}_0$. By the same token, the center of $\mathcal{R}_0'$ is contained in that of $\mathcal{R}$. Uniqueness of the (normalized) dimension function now implies that D_0 is the restriction of D to $\mathcal{R}_0$; and D' is the restriction of D_0' to $\mathcal{R}'$.

Let E be a projection in $\mathcal{R}$ and y be Ex. Then $E = [\mathcal{R}'y]$, so that $D(E) = D'([\mathcal{R}y])$. Since $[\mathcal{R}y] \in \mathcal{R}' \subseteq \mathcal{R}_0'$, $D(E) = D'([\mathcal{R}y]) = D_0'([\mathcal{R}y]) \geq D_0'([\mathcal{R}_0y]) = D_0([\mathcal{R}_0'y]) = D([\mathcal{R}_0'y]) \geq D([\mathcal{R}'y]) = D(E)$. Thus $D(E) = D([\mathcal{R}_0'y])$; and, since $E \leq [\mathcal{R}_0'y]$, $E = [\mathcal{R}_0'y] \in \mathcal{R}_0$. Hence each projection in $\mathcal{R}$ lies in $\mathcal{R}_0$; and $\mathcal{R} = \mathcal{R}_0$, contradicting the hypotheses.

[12] This is a consequence of Lemma 9.3.3 of reference 7 (as in reference 9, p. 242, Proposition 3, or Lemma 3.3.4 of reference 13).

[13] R. Kadison, Ann. Math. **66**, 304 (1957), see Chap. III.

[14] See reference 13, Theorem 3.3.8.

IV. TENSOR PRODUCTS

Lemma 2 is the key to[9]

Theorem 3. If $\mathcal{R}_1$ and $\mathcal{R}_2$ are semifinite von Neumann algebras, then $(\mathcal{R}_1 \otimes \mathcal{R}_2)' = \mathcal{R}_1' \otimes \mathcal{R}_2'$.

For the proof of this, we shall want:

Lemma 4. If $\mathcal{R}_0$ and $\mathcal{R}$ are von Neumann algebras such that $\mathcal{R}_0 \subseteq \mathcal{R}$, the center of $\mathcal{R}$ is contained in that of $\mathcal{R}_0$, and $\{E_\alpha'\}$ is a family of projections in $\mathcal{R}'$ with union I such that $\mathcal{R}E_\alpha' = \mathcal{R}_0E_\alpha'$ (or, dually, $E_\alpha'\mathcal{R}'E_\alpha' = E_\alpha'\mathcal{R}_0'E_\alpha'$) for each α; then $\mathcal{R}_0 = \mathcal{R}$.

Proof: Since von Neumann algebras are generated by their projections, it suffices to show that each projection E in $\mathcal{R}$ lies in $\mathcal{R}_0$. By assumption, for each α there is an A_0 in $\mathcal{R}_0$ such that $EE_\alpha' = A_0E_\alpha'$. Let F_0 be the range projection of A_0. Then F_0 lies in $\mathcal{R}_0$.[15] Now F_0E_α' ($= E_\alpha'F_0$) and A_0E_α' ($= E_\alpha'A_0$) are both projections with $\{E_\alpha'A_0x\}$ dense in their ranges; so that $A_0E_\alpha' = F_0E_\alpha' = EE_\alpha'$. With T' in $\mathcal{R}'$, $T'EE_\alpha' = ET'E_\alpha' = T'F_0E_\alpha' = F_0T'E_\alpha'$; so that $EP_\alpha = F_0P_\alpha$, where P_α is the central carrier of E_α' (relative to $\mathcal{R}'$).[16] Since the center of $\mathcal{R}$ is contained in the center of $\mathcal{R}_0$, F_0P_α lies in $\mathcal{R}_0$. Moreover,

$$E(\bigvee_\alpha P_\alpha) \geq E(\bigvee_\alpha E_\alpha') = E\cdot I = E,$$

so that

$$E = E(\bigvee_\alpha P_\alpha) = \bigvee_\alpha EP_\alpha = \bigvee_\alpha F_0P_\alpha$$

lies in $\mathcal{R}_0$.

If $E_\alpha'\mathcal{R}'E_\alpha' = E_\alpha'\mathcal{R}_0'E_\alpha'$, then $\mathcal{R}E_\alpha' = \mathcal{R}_0E_\alpha'$ for each α[17]; and from the preceding, $\mathcal{R} = \mathcal{R}_0$.

Proof of Theorem 3: With A_1' in $\mathcal{R}_1'$ and A_2' in $\mathcal{R}_2'$, $A_1' \otimes A_2'$ commutes with each $A_1 \otimes A_2$ in $\mathcal{R}_1 \otimes \mathcal{R}_2$ so that $A_1' \otimes A_2'$ lies in $(\mathcal{R}_1 \otimes \mathcal{R}_2)'$. Thus $\mathcal{R}_1' \otimes \mathcal{R}_2' \subseteq (\mathcal{R}_1 \otimes \mathcal{R}_2)'$. The problem resides in establishing the reverse inclusion.

Suppose E_1' and E_2' are projections in $\mathcal{R}_1'$, $\mathcal{R}_2'$, respectively, such that

$$[(\mathcal{R}_1E_1') \otimes (\mathcal{R}_2E_2')]' = (\mathcal{R}_1E_1')' \otimes (\mathcal{R}_2E_2')'. \qquad (4.1)$$

Then

$$(E_1' \otimes E_2')(\mathcal{R}_1 \otimes \mathcal{R}_2)'(E_1' \otimes E_2') = (E_1'\mathcal{R}_1'E_1') \otimes (E_2'\mathcal{R}_2'E_2') = (E_1' \otimes E_2')(\mathcal{R}_1' \otimes \mathcal{R}_2')(E_1' \otimes E_2'). \qquad (4.2)$$

With $\mathcal{R}_1$ and $\mathcal{R}_2$ Abelian, E_1', E_2' as above and cyclic; $\mathcal{R}_1E_1'$, $\mathcal{R}_2E_2'$ and $(\mathcal{R}_1E_1') \otimes (\mathcal{R}_2E_2')$ are maximal Abelian since each is Abelian and has a cyclic

[15] The range projection F_0 commutes with R'. Cf. J. von Neumann, Math. Ann. **102**, 370 (1929).

[16] See reference 13, especially Sec. 3.1.

[17] See reference 7, Lemma 11.3.2, and reference 9, p. 18, Proposition 1.

vector.[18] Thus (4.1), and, hence, (4.2) hold, in this case. Since the union of projections $E_1' \otimes E_2'$ in $\mathcal{R}_1' \otimes \mathcal{R}_2'$, with E_1', E_2' cyclic, is I; $(\mathcal{R}_1 \otimes \mathcal{R}_2)' = \mathcal{R}_1' \otimes \mathcal{R}_2'$, from Lemma 4, when $\mathcal{R}_1$ and $\mathcal{R}_2$ are Abelian—once we note that $\mathcal{R}_1 \otimes \mathcal{R}_2$, being Abelian, is its own center as well as that of $(\mathcal{R}_1 \otimes \mathcal{R}_2)'$ and is contained in $\mathcal{R}_1' \otimes \mathcal{R}_2'[\subseteq (\mathcal{R}_1 \otimes \mathcal{R}_2)']$ and hence in the center of $\mathcal{R}_1' \otimes \mathcal{R}_2'$.

For arbitrary von Neumann algebras $\mathcal{R}_1$, $\mathcal{R}_2$ with centers $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively, the center $\mathcal{C}$ of $\mathcal{R}_1 \otimes \mathcal{R}_2$ is $\mathcal{C}_1 \otimes \mathcal{C}_2$. In fact, $\mathcal{C}_1 \otimes \mathcal{C}_2 \subseteq \mathcal{C}$; while $\mathcal{R}_1 \otimes I \subseteq \mathcal{R}_1 \otimes \mathcal{R}_2 \subseteq \mathcal{C}'$ and $\mathcal{R}_1' \otimes I \subseteq \mathcal{R}_1' \otimes \mathcal{R}_2' \subseteq (\mathcal{R}_1 \otimes \mathcal{R}_2)' \subseteq \mathcal{C}'$. Now, $\mathcal{R}_1$ and $\mathcal{R}_1'$ generate $\mathcal{C}_1'$; so that $\mathcal{C}_1' \otimes I \subseteq \mathcal{C}'$. Similarly $I \otimes \mathcal{C}_2' \subseteq \mathcal{C}'$. Thus $\mathcal{C}_1' \otimes \mathcal{C}_2' = (\mathcal{C}_1 \otimes \mathcal{C}_2)' \subseteq \mathcal{C}'$; and $\mathcal{C}_1 \otimes \mathcal{C}_2 \supseteq \mathcal{C}$. It follows that $\mathcal{C}_1 \otimes \mathcal{C}_2 = \mathcal{C}$. We conclude that $\mathcal{R}_1' \otimes \mathcal{R}_2'$ and $(\mathcal{R}_1 \otimes \mathcal{R}_2)'$ have the same center (viz. $\mathcal{C}_1 \otimes \mathcal{C}_2$, the center of $\mathcal{R}_1 \otimes \mathcal{R}_2$).

Combining this last conclusion with (4.1), (4.2), the comment that $P(\vee_\gamma G_\gamma) = \vee_\gamma PG_\gamma$ when $PG_\gamma = G_\gamma P$ for each γ, and Lemma 4, we see that it suffices to prove

$$[(\mathcal{R}_1 E_\alpha') \otimes (\mathcal{R}_2 F_\beta')]' = (\mathcal{R}_1 E_\alpha')' \otimes (\mathcal{R}_2 F_\beta')', \tag{4.3}$$

for all α and β, where $\{E_\alpha'\}$ and $\{F_\beta'\}$ are families of projections in $\mathcal{R}_1'$ and $\mathcal{R}_2'$, respectively, with union I. With $\mathcal{R}_1$ and $\mathcal{R}_2$ semifinite, $\mathcal{R}_1'$ and $\mathcal{R}_2'$ are[19]; and each is generated by its finite cyclic projections. If E' and F' are finite cyclic projections in $\mathcal{R}_1'$ and $\mathcal{R}_2'$, respectively, $(\mathcal{R}_1 E')'(= E' R_1' E')$ and $(\mathcal{R}_2 F')'$ are finite; and their commutants have cyclic vectors. We may assume, therefore, that $\mathcal{R}_1'$ and $\mathcal{R}_2'$ are finite; and that $\mathcal{R}_1$ and $\mathcal{R}_2$ have cyclic vectors.

Since $(\mathcal{R}_1 \otimes \mathcal{R}_2)' = \mathcal{R}_1' \otimes \mathcal{R}_2'$ is equivalent to $\mathcal{R}_1 \otimes \mathcal{R}_2 = (\mathcal{R}_1' \otimes \mathcal{R}_2')'$, and the finite cyclic projections in $\mathcal{R}_1$, $\mathcal{R}_2$ have union I, it suffices to prove $(\mathcal{R}_1' E)' \otimes (\mathcal{R}_2' F)' = [(\mathcal{R}_1' E) \otimes (\mathcal{R}_2' F)]'$, for all such projections E and F. But now $(\mathcal{R}_1' E)'$, $(\mathcal{R}_2' F)'$, $\mathcal{R}_1' E$, and $\mathcal{R}_2' F$ are all finite and $\mathcal{R}_1' E$, $\mathcal{R}_2' F$ have cyclic vectors. We may assume that $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_1'$, and $\mathcal{R}_2'$ are finite and $\mathcal{R}_1$, $\mathcal{R}_2$ have cyclic vectors x and y, respectively. For each vector z, $D_1([\mathcal{R}_1' z]) \leq D_1([\mathcal{R}_1' x])$.[14] But $D_1(I - [\mathcal{R}_1' z]) = I - D_1([\mathcal{R}_1' z]) \geq D_1([\mathcal{R}_1' x]) - D_1([\mathcal{R}_1' z])$, so that there is a partial isometry V in $\mathcal{R}_1$ with initial space $[\mathcal{R}_1' x]$ and final space $V([\mathcal{R}_1' x]) = [\mathcal{R}_1' Vx]$ containing $[\mathcal{R}_1' z]$. Now $[\mathcal{R}_1 Vx] \supseteq [\mathcal{R}_1 V^* Vx] = [\mathcal{R}_1 x]$; so that each cyclic projection in $\mathcal{R}_1$ is contained in a projection $[\mathcal{R}_1' w]$, with w cyclic for $\mathcal{R}_1$. Hence the union of such projections in $\mathcal{R}_1$ is I. Since the same is true for $\mathcal{R}_2$, it suffices to prove $(\mathcal{R}_1'[\mathcal{R}_1' x] \otimes \mathcal{R}_2'[\mathcal{R}_2' y])' = ([\mathcal{R}_1' x]\mathcal{R}_1[\mathcal{R}_1' x]) \otimes ([\mathcal{R}_2' y]\mathcal{R}_2[\mathcal{R}_2' y])$, for all cyclic vectors x for $\mathcal{R}_1$ and y for $\mathcal{R}_2$. But $\mathcal{R}_1'[\mathcal{R}_1' x]$ and $[\mathcal{R}_1' x]\mathcal{R}_1[\mathcal{R}_1' x]$ are finite with x as cyclic vector for each, while $\mathcal{R}_2'[\mathcal{R}_2' y]$ and $[\mathcal{R}_2' y]\mathcal{R}_2[\mathcal{R}_2' y]$ are finite with y as cyclic vector for each.

We may assume $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_1'$, $\mathcal{R}_2'$ are finite with x a cyclic vector for $\mathcal{R}_1$, $\mathcal{R}_1'$, and y a cyclic vector for $\mathcal{R}_2$, $\mathcal{R}_2'$. In this case, $x \otimes y$ is cyclic for $\mathcal{R}_1 \otimes \mathcal{R}_2$ and $\mathcal{R}_1' \otimes \mathcal{R}_2'$ $[\subseteq (\mathcal{R}_1 \otimes \mathcal{R}_2)']$; hence for $(\mathcal{R}_1 \otimes \mathcal{R}_2)'$. The product of the center-valued traces[20] on $\mathcal{R}_1$ and $\mathcal{R}_2$ extends to a (finite) center-valued trace on $\mathcal{R}_1 \otimes \mathcal{R}_2$,[21] so that $\mathcal{R}_1 \otimes \mathcal{R}_2$ is finite. Since $\mathcal{R}_1 \otimes \mathcal{R}_2$ has a cyclic vector, $(\mathcal{R}_1 \otimes \mathcal{R}_2)'$ is finite. From Lemma 2, $\mathcal{R}_1' \otimes \mathcal{R}_2' = (\mathcal{R}_1 \otimes \mathcal{R}_2)'$.

Remark. The formula for $(\mathcal{R}_1 \otimes \mathcal{R}_2)'$ has not been proved for $\mathcal{R}_1$ and $\mathcal{R}_2$ factors of type III.

V. JOINT CYCLIC AND SEPARATING VECTOR—REDUCTION OF THE PROBLEM

The presumption that the cyclic and separating vector of (1.1) does not exist can be cast as a conjecture in many forms. Two variants of this due to Guénin and Misra[6] are listed as:

B_1: If $\mathcal{R}_1$ is a proper subfactor of $\mathcal{R}$, both are factors of type I_∞, and ψ is a separating and cyclic vector for both $\mathcal{R}_1$ and $\mathcal{R}$, then each minimal projection in $\mathcal{R}_1$ is finite relative to $\mathcal{R}$.

B_2: If $\mathcal{R}_1$ is a proper subfactor of $\mathcal{R}$ unitarily equivalent to $\mathcal{R}$, ψ is a separating and cyclic vector for both $\mathcal{R}_1$ and $\mathcal{R}$, and $\mathcal{R}$ is the von Neumann algebra generated by $\mathcal{R}_1$ and $\mathcal{R}_1' \cap \mathcal{R}$; then each finite projection in $\mathcal{R}_1$ is finite relative to $\mathcal{R}$.

Under the hypothesis of B_1, $\mathcal{R}_1' \cap \mathcal{R}$ is a factor of type I_n (n possibly ∞). The dimension of a minimal projection in $\mathcal{R}_1$ relative to $\mathcal{R}$ is n. To see this, note that the situation does not change if we replace $\mathcal{R}$ by a von Neumann algebra isomorphic to it. Assume, for the moment, that $\mathcal{R}$ is all bounded operators on some (separable) Hilbert space—so that $\mathcal{R}_1$ is then a I_∞ factor on this space with I_n commutant $\mathcal{R}_1'$ $(= \mathcal{R}_1' \cap \mathcal{R})$. If E is a minimal projection in $\mathcal{R}_1$, the mapping $A_1' \to A_1' E$ is an isomorphism (since $\mathcal{R}_1'$ is a factor) of $\mathcal{R}_1'$ onto the algebra of all bounded operators acting on E (by minimality of E)— which algebra is, accordingly, of type I_n. Thus E is n-dimensional (with $\mathcal{R}$ all bounded operators), i.e., E has dimension n relative to $\mathcal{R}$.

Conjecture B_1 becomes then: $\mathcal{R}_1' \cap \mathcal{R}$ cannot be of type I_∞ with ψ a cyclic and separating vector

[18] See reference 15. This can be made to follow from reference 7, Lemma 9.3.3, or reference 9, p. 242, Proposition 3.
[19] This follows from the references of 12, or explicitly in reference 9, p. 101, Corollaire 1.
[20] See reference 9, p. 267, Théorème 3, or R. Kadison, Proc. Am. Math. Soc. **12**, 973 (1961).
[21] See reference 9, p. 56, Théorème 2.

for the factor $\mathcal{R}$ of type I_∞ with type I_∞ commutant, and for the subfactor $\mathcal{R}_1$ of type I_∞ of $\mathcal{R}$—i.e., B_1 asserts that (1.1) has a negative answer. Now if $\mathcal{R}$ and $\mathcal{S}$ are factors of type I_∞ (on separable Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$) each with commutant of type I_∞, each has a separating and cyclic vector[22] and they are unitarily equivalent[23]; viz. there is a unitary transformation U of $\mathcal{H}$ onto $\mathcal{K}$ such that the mapping $A \to UAU^{-1}$ of bounded operators on $\mathcal{H}$ into bounded operators on $\mathcal{K}$ maps $\mathcal{R}$ *-isomorphically onto $\mathcal{S}$. If $\mathcal{R}_1$ and $\mathcal{S}_1$ are subfactors of $\mathcal{R}$ and $\mathcal{S}$, respectively, of type I_∞, each with commutant relative to $\mathcal{R}$ and $\mathcal{S}$($\mathcal{R}_1' \cap \mathcal{R}$ and $\mathcal{S}_1' \cap \mathcal{S}$) of type I_∞, then each has absolute commutant of type I_∞ and so has its own cyclic and separating vector, from the preceding remarks. Moreover, $U\mathcal{R}_1U^{-1}$ is a type I_∞ subfactor of $\mathcal{S}$ with relative commutant $(U\mathcal{R}_1U^{-1})' \cap \mathcal{S}$ of type I_∞. Again, from the preceding remarks (representing $\mathcal{S}$ as all bounded operators on some separable space), there is a unitary operator V in $\mathcal{S}$ such that $VU\mathcal{R}_1U^{-1}V^{-1} = \mathcal{S}_1$. Thus VU is a unitary transformation of $\mathcal{H}$ onto $\mathcal{K}$ carrying $\mathcal{R}$ onto $\mathcal{S}$, $\mathcal{R}_1$ onto $\mathcal{S}_1$, and, hence a separating and cyclic vector for $\mathcal{R}$ and $\mathcal{R}_1$, if one exists, onto such a vector for $\mathcal{S}$ and $\mathcal{S}_1$. Thus, if one such pair $\mathcal{R}$, $\mathcal{R}_1$ has a joint separating and cyclic vector, all such pairs do (all being unitarily equivalent to $\mathcal{R}$ and $\mathcal{R}_1$):

We have noted that each of $\mathcal{R}$ and $\mathcal{R}_1$ has its own cyclic and separating vector. The problem is whether one vector will serve as such for both of them. Suppose x is such a vector. In any event, $\mathcal{R}$ and $\mathcal{R}_1$, being of type I_∞ with (absolute) commutant of type I_∞, are unitarily equivalent, as noted above. Further, $\mathcal{R}$ and $\mathcal{R}_1$ being factors of type I_∞ implies[24] that $\mathcal{R}$ is unitarily equivalent to the tensor product of $\mathcal{R}_1$ and $\mathcal{R}_1' \cap \mathcal{R}$—in particular, $\mathcal{R}$ is generated by $\mathcal{R}_1$ and $\mathcal{R}_1' \cap \mathcal{R}$ [and $(\mathcal{R}_1' \cap \mathcal{R})' \cap \mathcal{R} = \mathcal{R}_1$]. As noted, the minimal projections of $\mathcal{R}_1$, which are certainly finite in $\mathcal{R}_1$, have dimension ∞ relative to $\mathcal{R}$, with $\mathcal{R}_1' \cap \mathcal{R}$ of type I_∞. Thus the example constructed in this and the next section, to show that (1.1) has an affirmative answer, settles both conjectures B_1 and B_2 negatively.

We begin by constructing a factor $\mathcal{R}$ of type I_∞ and a subfactor $\mathcal{R}_1$ of type I_∞ with $\mathcal{R}_1' \cap \mathcal{R}$ of type I_∞ (which pair will be a "canonical form" for all pairs, by virtue of the preceding remarks). Let $\mathcal{H}$ be a (fixed) separable Hilbert space, $\mathcal{B}(\mathcal{H})$ the algebra of all bounded operators on $\mathcal{H}$, $\mathcal{H}'$ the direct sum $\mathcal{H} \oplus \mathcal{H} \oplus \cdots$ of $\mathcal{H}$ with itself a countable number of times, and $\mathcal{H}''$ the same, with $\mathcal{H}'$ in place of $\mathcal{H}$. With T an operator on $\mathcal{H}$, let $T^\sim$ be the operator on $\mathcal{H}'$ defined by $T^\sim(x') = (Tx_1, Tx_2, \cdots)$, where x' [$= (x_1, x_2, \cdots)$] is a vector in $\mathcal{H}'$. Similarly, if $\bar{T}$ is an operator on $\mathcal{H}'$, we can associate with it an operator $\bar{T}^\sim$ on $\mathcal{H}''$. In terms of (infinite) matrices with operator entries, $T^\sim$ is the matrix with all off-diagonal entries 0 and each diagonal entry equal to T. Viewed as infinite (operator entry) matrices, the operators on $\mathcal{H}''$ are infinite matrices each of whose entries is an infinite matrix with entries operators on $\mathcal{H}$. Thus $\mathcal{B}(\mathcal{H})^{\sim\sim}$ is an "infinite copy" of $\mathcal{B}(\mathcal{H})^\sim$; and $\mathcal{B}(\mathcal{H}')^\sim$, an infinite copy of $\mathcal{B}(\mathcal{H}')$, contains $\mathcal{B}(\mathcal{H})^{\sim\sim}$. Both are factors of type I_∞ with commutants of type I_∞. Denote $\mathcal{B}(\mathcal{H})^{\sim\sim}$ by $\mathcal{R}_1$ and $\mathcal{B}(\mathcal{H}')^\sim$ by $\mathcal{R}$. The matrices representing operators in $(\mathcal{B}(\mathcal{H}')^\sim)'$ have scalar multiples of the identity operator on $\mathcal{H}'$ as entries. Moreover, $(\mathcal{B}(\mathcal{H})^{\sim\sim})' \cap \mathcal{B}(\mathcal{H}')^\sim$ ($= \mathcal{R}_1' \cap \mathcal{R}$) consists of operators whose matrix representation has each principal (diagonal) infinite matrix block [i.e., operator in $\mathcal{B}(\mathcal{H}')$] equal to one infinite matrix, all of whose entries are scalar multiples of the identity operator on $\mathcal{H}$, and all nonprincipal blocks equal to 0. Thus $\mathcal{R}_1' \cap \mathcal{R}$ is and infinite copy of a factor of type I_∞ [viz. $(\mathcal{B}(\mathcal{H})^\sim)'$], and is itself a factor of type I_∞.

In the notation of tensor products of Hilbert spaces, $\mathcal{H}'$ can be identified with $\mathcal{H} \otimes \mathcal{H}$, and $\mathcal{H}''$ with $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$, $\mathcal{B}(\mathcal{H})^\sim$ with $\mathcal{B}(\mathcal{H}) \otimes I$, $\mathcal{B}(\mathcal{H}')$with $\mathcal{B}(\mathcal{H}) \otimes \mathcal{B}(\mathcal{H})$, $\mathcal{B}(\mathcal{H})^{\sim\sim}$ ($= \mathcal{R}_1$) with $\mathcal{B}(\mathcal{H}) \otimes I \otimes I$, $\mathcal{B}(\mathcal{H}')^\sim (= \mathcal{R})$ with $\mathcal{B}(\mathcal{H}) \otimes \mathcal{B}(\mathcal{H}) \otimes I (= \mathcal{B}(\mathcal{H}') \otimes I)$, and $\mathcal{R}_1' \cap \mathcal{R}$ with $I \otimes \mathcal{B}(\mathcal{H}) \otimes I$.

In the development which follows, we shall derive conditions on the set of coordinates of a vector x' in $\mathcal{H}'$ under which it is a separating vector for $\mathcal{B}(\mathcal{H})^\sim$ and conditions under which it is a cyclic vector for $\mathcal{B}(\mathcal{H})^\sim$.

Definition 5. A set of vectors $\{x_j\}$ *in* $\mathcal{H}$ *is said to be an* L_2 *set when* $\sum_{j=1}^{\infty} \|x_j\|^2 < \infty$. *An* L_2 *set of vectors* $\{x_j\}$ *in* $\mathcal{H}$ *will be said to be* L_2*-independent when* $\sum_{j=1}^{\infty} \alpha_j x_j = 0$, *for* α_j *with* $\sum_{j=1}^{\infty} |\alpha_j|^2 < \infty$, *implies* $\alpha_j = 0$ *for all* j.

Remark 6. The L_2 sets are precisely the possible sets of coordinates of vectors in $\mathcal{H}'$.

Remark 7. Note that with $\{x_j\}$ an L_2 set, and $\sum_{j=1}^{\infty} |\alpha_j|^2 < \infty$, $\sum_{j=1}^{\infty} \alpha_j x_j$ converges absolutely, for $\sum_{j=1}^{\infty} |\alpha_j| \, \|x_j\| \leq (\sum_{j=1}^{\infty} |\alpha|^2)^{\frac{1}{2}} (\sum_{j=1}^{\infty} \|x_j\|^2)^{\frac{1}{2}}$ (by Cauchy–Schwarz).

Lemma 8. The set of vectors $\{x_j\}$ *in* $\mathcal{H}$ *is* L_2*-independent if and only if there exists a Hilbert–Schmidt operator* T *on* $\mathcal{H}$ *which is one–one* [i.e., *null*

[22] See reference 7, p. 182, Theorem X; or E. L. Griffin, Jr., Trans. Am. Math. Soc. 75, 471 (1953), especially Lemma 1.2.8; or reference 13, Lemma 3.3.6.

[23] See reference 9, p. 233, Théorème 3.

[24] See reference 7, Lemma 3.2.4.

space (0)] and an orthonormal basis $\{y_j\}$ *for* $\mathcal{H}$ *such that* $Ty_j = x_j$, *for all j.*

Proof: By a unitary equivalence, we may assume that $\mathcal{H}$ is l_2 (sequence Hilbert space) and $x_j = (\alpha_{1j}, \alpha_{2j}, \cdots)$. Let T be the operator on $\mathcal{H}$ (so represented) which corresponds to the matrix (α_{kj}) relative to the orthonormal basis $\{y_j\}$, where y_j has jth coordinate 1 and all other coordinates 0. Then T is a Hilbert–Schmidt operator if and only if $\{x_j\}$ is an L_2 set, for $\sum_{j=1}^{\infty} ||x_j||^2 = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} |\alpha_{kj}|^2 =$ Trace (T^*T).

With $\{x_j\}$ an L_2 set, and $z = (\beta_1, \beta_2, \cdots)$, $0 = Tz = (\sum_{j=1}^{\infty} \beta_j\alpha_{1j}, \sum_{j=1}^{\infty} \beta_j\alpha_{2j}, \cdots)$, if and only if $\sum_{j=1}^{\infty} \beta_j\alpha_{kj} = 0$, for all k. This last is the case, if and only if $\sum_{j=1}^{\infty} \beta_jx_j = 0$. Thus T is one–one if and only if $\{x_j\}$ is L_2-independent.

Lemma 9. The vector $x' = (x_1, x_2, \cdots)$ *in* $\mathcal{H}'$ *is cyclic for* $\mathcal{B}(\mathcal{H})^{\sim}$ *if and only if* $\{x_j\}$ *is* L_2*-independent.*

For the proof of this, we shall need the following two remarks:

Remark 10. If $\bar{T}$ in $\mathcal{B}(\mathcal{H}')$ has $\alpha_{kj}I$ as k, jth entry, and z' in $\mathcal{H}'$ has all coordinates 0 except the jth, which is some unit vector z in $\mathcal{H}$, then $\bar{T}z' = (\alpha_{1j}z, \alpha_{2j}z, \cdots)$; so that $||\bar{T}z'||^2 = \sum_{k=1}^{\infty} |\alpha_{kj}|^2 \leq ||\bar{T}||^2\ ||z'||^2 = ||\bar{T}||^2$. Thus all columns of this special $\bar{T}$ are "square summable". Applying this to $\bar{T}^*$, we conclude that all rows of $\bar{T}$ are "square summable".

Remark 11. If $\bar{T}$ has β_jI as entry in the first row and jth column, and 0 at all other entries, where $\beta^2 = \sum_{j=1}^{\infty} |\beta_j|^2 < \infty$, then, with $z' = (z_1, z_2, \cdots)$, $\bar{T}z' = (\sum_{j=1}^{\infty} \beta_jz_j, 0, 0, \cdots)$. Thus

$$||\bar{T}z'|| = \left|\left| \sum_{j=1}^{\infty} \beta_jz_j \right|\right| \leq \sum_{j=1}^{\infty} |\beta_j|\ ||z_j||$$

$$\leq \left(\sum_{j=1}^{\infty} |\beta_j|^2\right)^{\frac{1}{2}} \left(\sum_{j=1}^{\infty} ||z_j||^2\right)^{\frac{1}{2}} = \beta\ ||z'||,$$

which establishes both the convergence of $\sum_{j=1}^{\infty} \beta_jz_j$, so that $\bar{T}$ is a well-defined linear operator on $\mathcal{H}'$, and the boundedness of $\bar{T}$.

Proof of Lemma 9: The vector x' is cylic for $\mathcal{B}(\mathcal{H})^{\sim}$ if and only if it is separating for $(\mathcal{B}(\mathcal{H})^{\sim})'$.[25] From our earlier comments about the matrix form of an operator $\bar{T}$ in $(\mathcal{B}(\mathcal{H})^{\sim})'$, we know that the k, jth entry is $\alpha_{kj}I$, with α_{kj} some scalar. Thus $\bar{T}x' = 0$, if and only if $\sum_{j=1}^{\infty} \alpha_{kj}x_j = 0$, for all k. From Remark 10, $\sum_{j=1}^{\infty} |\alpha_{kj}|^2 < \infty$, for all k. Thus if $\{x_j\}$ is L_2-independent, $\alpha_{kj} = 0$, for all k, j, $\bar{T} = 0$; and x' is separating for $(\mathcal{B}(\mathcal{H})^{\sim})'$ and cyclic for $\mathcal{B}(\mathcal{H})^{\sim}$.

[25] See reference 9, p. 6, Proposition 5 (note: "totalisateur" replaces "cyclic").

On the other hand, if x' is cyclic for $\mathcal{B}(\mathcal{H})^{\sim}$, so, separating for $(\mathcal{B}(\mathcal{H})^{\sim})'$, and $\sum_{j=1}^{\infty} \beta_jx_j = 0$, with $\sum_{j=1}^{\infty} |\beta_j|^2 < \infty$, then $\bar{T}$, with β_jI as 1, jth entry and all entries 0 in rows other than the first, is bounded, by Remark 11. Hence $\bar{T}$ lies in $(\mathcal{B}(\mathcal{H})^{\sim})'$ (by virtue of its matrix form). But $\bar{T}x' = (\sum_{j=1}^{\infty} \beta_jx_j, 0, 0, \cdots) = 0$; so that $\bar{T} = 0$ and $\beta_j = 0$, for all j. Thus $\{x_j\}$ is L_2-independent.

Lemma 12. The vector $x' = (x_1, x_2, \cdots)$ *is separating for* $\mathcal{B}(\mathcal{H})^{\sim}$ *if and only if its set of coordinates* $\{x_k\}$ *spans* $\mathcal{H}$.

Proof: We have $T^{\sim}x' = 0$ if and only if $Tx_k = 0$, for all k, which is the case if and only if T annihilates the subspace M of $\mathcal{H}$ spanned by $\{x_k\}$. Now T annihilating M is equivalent to T (and hence $T^{\sim}$) being 0, if and only if $M = \mathcal{H}$.

From Lemmas 9 and 12, we see that $\mathcal{R}_1$ and $\mathcal{R}$ have a joint cyclic and separating vector if and only if there is an L_2 set $\{x_j'\}$ in $\mathcal{H}'$ which spans $\mathcal{H}'$—so that $x'' = (x_1', x_2', \cdots)$ in $\mathcal{H}''$ is separating for $\mathcal{R}$ (and *a fortiori* for $\mathcal{R}_1$)—such that $\{x_{kj}\}$ is L_2-independent, where $x_k' = (x_{k1}, x_{k2}, \cdots)$—so that x'' is cyclic for $\mathcal{R}_1$ (and *a fortiori* for $\mathcal{R}$).

It is useful to view the desired construction in intrinsic form (say in our fixed Hilbert space $\mathcal{H}$). We ask for a sequence $E_1, E_2, \cdots$ of mutually orthogonal, infinite-dimensional subspaces of $\mathcal{H}$, an isometry V_j of E_j onto E_1, and an L_2 set $\{x_k\}$ which spans $\mathcal{H}$ such that $\{V_jE_jx_k\}$ is L_2-independent (in E_1). In this formulation, E_1 replaces $\mathcal{H}$, $\mathcal{H}$ replaces $\mathcal{H}'$ (as a direct sum of the E_j or E_1 with itself a countable number of times by virtue of the isometric identification V_j of E_j with E_1), x_k replaces x_k', and $V_jE_jx_k$ replaces x_{kj}. It is in this form that we establish the existence of a joint separating and cyclic vector, in the next section.

VI. THE CONSTRUCTION

We state the result being proved explicitly as:

Theorem 13. If $\mathcal{R}$ *is a factor of type* I_∞ *acting on the separable Hilbert space* $\mathcal{H}$, $\mathcal{R}'$ *is of type* I_∞ *and* $\mathcal{R}_1$ *is a subfactor of* $\mathcal{R}$ *of type* I_∞ *with relative commutant* $\mathcal{R}_1' \cap \mathcal{R}$ *a factor of type* I_∞, *then there is a vector x in* $\mathcal{H}$ *which is cyclic and separating for both* $\mathcal{R}_1$ *and* $\mathcal{R}$.

Proof: For the purposes of this construction (and from the discussion of the preceding section), we may take $\mathcal{H}$ in the specific representation $L_2([0, 1])$ (relative to Lebesgue measure). Following the required construction as outlined at the end of the last section, we take x_k to be the function $\gamma \to \gamma^k/k$ (actually, the equivalence class of all square-summable functions which differ from this function at most on a set of measure 0). As E_k, we choose the subspace of $\mathcal{H}$ consisting of those functions which vanish almost everywhere (a.e.) outside of $[2^{-k}, 2^{-(k-1)}]$. Let $\bar{x}_k = kx_k$.

Note that the transformation U_k' defined by $(U_k'f)(\gamma) = 2^{-(k-1)/2}f(\gamma/2^{k-1})$, for continuous f in E_k, maps this set of functions isometrically onto the set of continuous functions in E_1. Denote by U_k the (unique) extension of U_k' to E_k mapping E_k isometrically onto E_1. Note also that $U_jE_jx_k = k^{-1}2^{-(j-1)(k+\frac{1}{2})}E_1\bar{x}_k$. Let W_k be the operator on $\mathcal{H}$ defined by $W_kf = f_k\cdot f$, for f in $\mathcal{H}$, where f_k is 0 on $[0, \frac{1}{2}]$, 1 on $[\frac{1}{2}, 1-2^{-(k+1)})$, and -1 on $[1-2^{-(k+1)}, 1]$. We note that each W_k maps E_1 isometrically onto itself. Finally, we take V_k to be W_kU_k.

To see that the choices satisfy the desired conditions, observe that $\{x_k\}$ spans $\mathcal{H}$ by virtue of the Weierstrass Polynomial Approximation Theorem. Suppose $\sum_{j,k=1}^{\infty} \alpha_{jk}V_jE_jx_k = 0$, with $\sum_{j,k=1}^{\infty} |\alpha_{jk}|^2 < \infty$. Then $0 = \sum_{j,k=1}^{\infty} \alpha_{jk}k^{-1}2^{-(j-1)(k+\frac{1}{2})}W_jE_1\bar{x}_k = \sum_{j,k=1}^{\infty} \beta_{jk}W_jE_1\bar{x}_k$, with $\sum_{j,k=1}^{\infty} |\beta_{jk}| < \infty$. Thus $0 = \sum_{j=1}^{\infty} W_jy_j$, where $y_j = \sum_{k=1}^{\infty} \beta_{jk}E_1\bar{x}_k$. Now y_j is the (equivalence class of the) restriction to $[\frac{1}{2}, 1]$ of g_j, where

$$g_j(\gamma) = \sum_{k=1}^{\infty} \beta_{jk}\gamma^k, \tag{6.1}$$

so that g_j is analytic on the open unit disk $\mathfrak{D}$ in the plane of complex numbers (since $\sum_{k=1}^{\infty} |\beta_{jk}| < \infty$). Since f_jg_j is in the equivalence class W_jy_j,

$$\text{l.i.m.} \sum_{j=1}^{n} f_jg_j = 0 \tag{6.2}$$

(i.e., the sum $\sum_{j=1}^{\infty} f_jg_j$ converges in L_2 to 0). But f_jg_j is g_j on $[\frac{1}{2}, \frac{3}{4})$; so that $g = \sum_{j=1}^{\infty} g_j$ is 0 (a.e) on $[\frac{1}{2}, \frac{3}{4})$. Since g is analytic on $\mathfrak{D}$, g is 0 on $\mathfrak{D}$. Define g_0 to be 0; and note that f_k is -1 on the interval $[1 - 2^{-(k+1)}, 1 - 2^{-(k+2)})$ $(= a)$, while f_j is 1 on a, for $j = k+1, k+2, \cdots$. Suppose we have established that $g_0, \cdots, g_{k-1}$ are 0; so that $\sum_{j=k}^{\infty} g_j$ is 0. Then, from (6.2), $\lim [\sum_{j=k+1}^{n} g_j - g_k] = 0$ on a; so that $\lim \sum_{j=k}^{n} g_j = 2g_k$ on a. Since g_k is analytic on $\mathfrak{D}$, $g_k = 0$. By induction, each g_k is 0. From (6.1), $\beta_{jk} = 0$, for all j and k. It follows that $\{V_jE_jx_k\}$ is L_2-independent; and the proof is complete.

VII. REGIONS WITH FACTORS NOT OF TYPE *I* (ARAKI)

Araki[11] shows that the von Neumann algebra of local observables associated with a certain region is a factor not of type *I*. He considers the region $\mathcal{O}$ of space–time, the coordinates of whose points satisfy $|x_0| < |x_1|$, $x_1 > 0$, x_2, and x_3, arbitrary (and also the interior of the set of points spacelike with respect to these—for the purpose of the commutant). He notes that $\mathcal{O}$ is invariant under translations in x_2 and x_3, and that the unitary operators associated with such translations have the vacuum ψ_0 as unique invariant state. The von Neumann algebra $\mathcal{R}$ associated with $\mathcal{O}$ is a factor which has ψ_0 as separating and cyclic vector. From this data, we conclude that $\mathcal{R}$ is not of type *I*. In fact:

Proposition 14. If $\mathcal{R}$ is a factor acting on the Hilbert space $\mathcal{H}$, U is a unitary operator which induces a nontrivial automorphism of $\mathcal{R}$, ψ_0 is separating for $\mathcal{R}$, and ψ_0 spans the eigenspace for U corresponding to the eigenvalue 1, then $\mathcal{R}$ is not of type I.

Proof: If $\mathcal{R}$ is of type *I*, then $U = VW'$ with V in $\mathcal{R}$ and W' in $\mathcal{R}'$ (both unitary). (This is well-known: each automorphism of a type *I* factor is inner, as noted in Sec. V; and if V in $\mathcal{R}$ induces the same automorphism as U, then $W' = V^{-1}U$ commutes with $\mathcal{R}$.) Since $VW' = W'V$; U, V and W' commute. Thus $V\psi_0 = VU\psi_0 = UV\psi_0$, and by uniqueness, $V\psi_0 = a\psi_0$ with $|a| = 1$. Since ψ_0 is separating for $\mathcal{R}$, $V = aI$. Thus U $(= aW')$ is in $\mathcal{R}'$ and induces the trivial (identity) automorphism of $\mathcal{R}$.

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