REMOTE SENSING AS THE 'X-RAY CRYSTALLOGRAPHY' FOR URBAN 'DNA'

Alan Wilson

Centre for Advanced Spatial Analysis, University College London,
London WC1E 7HB

Corresponding Author's E-mail: a.g.wilson@ucl.ac.uk

Abstract— The aim of this paper is to establish a creative link between contemporary remote sensing and urban analysis. The argument is rooted in model-based urban analysis and it is shown that, mainly through spatial interaction modelling, the workings of the city – the 'physiology' - can be represented. Models have now been extended to have the capacity to represent structural dynamics and this is shown to be dependent on what can be characterised as 'urban DNA'. It is shown that urban information systems can be enhanced by remote sensing, particularly temporally, through the concept of an intelligent information warehouse (IIW) and that such a system relates to 'urban DNA' structure in the same way as X-ray crystallography relates to the elucidation of DNA. This adds to the potential capability of analysing, even predicting, urban phase transitions as an aid to planning.

1 INTRODUCTION

The key aim of urban analysis is to understand cities in depth, in part as a basis for more effective planning. This 'depth' is developed from contributions from many sources: different disciplines, different subdisciplines within geography. This paper focuses on the contribution of mathematical and computer modelling to this enterprise and seeks to link it to the potential of contemporary remote sensing. In order to illustrate what is a complicated argument in as simple a way as possible, the presentation is restricted to cities rather than the broader context of 'cities and regions' or, indeed, 'cities, regions and countries'. The extensions involve a different style of modelling, but the core argument on what we will call 'physiology' and 'DNA' remain the same (cf. Wilson, 2008-A).

We now have good descriptors and rich data sources as the foundations of model-based analysis – though it can be argued that these have been seriously under-deployed in urban geography – see Longley (2003). They can be assembled into information warehouses rooted in GIS technologies – these on a spectrum from hi-tech-hi-functionality to a kind of low-tech – e.g. Google Earth – with lower functionality but easy to use. Remote sensing is a key part of this in the mapping of the key physical features of urban structure. For an example relating to London, see Crooks (2008). Longley (2003), however, commented that "the core of urban geography has been abandoned....and even the inert representations of remote sensing seem to tell us more about urban systems than the intellectual core of the subject". His comment may in one sense be correct, but that is in part because urban geography has not taken advantage of the intellectual framework offered by modelling. It is this perspective which gives us a reasonable theoretical understanding of the functioning of cities – what might be called the 'physiology' – particularly through spatial interaction and location models. We now also have the beginnings of an understanding of how structure evolves – and we can then build on the 'physiology' analogy and connect this to a notion of urban 'DNA' (Wilson, 2008-C) as the basis for understanding the 'evolution' of cities. The full achievement of this final step in understanding represents one of the great contemporary scientific challenges – and also significant planning opportunities. The challenge of this paper is to explore what the "inert" representations of remote sensing can contribute to this challenge.

So what can remote sensing contribute? It is a major data source. But also, its products can be enriched by effective integration with other data sources. There are technical issues in how most effectively to achieve this, but it can be a key, through a related GIS, to organising the *intelligent* information warehouse – an IIW - which is needed to underpin urban analysis. It also has the advantage, through satellite technology, of being able to monitor development temporally more frequently than any other source.

These challenges can be elaborated and offer some interesting research problems but perhaps the biggest challenge relates to the evolution of urban structure, highlighted above as a grand challenge. The data that allowed the structure of DNA to be determined came from X-ray crystallography. Can remote sensing be the 'X-ray crystallography' for the decoding or urban 'DNA'? These questions are explored in subsequent sections. In sections 2 and 3, we use the urban retail system as an example to demonstrate the 'physiology' and 'DNA' concepts for a sub-system of a city. In section 4, we generalise this picture and show how it works on the basis of a more comprehensive urban model. In section 5, we review the design challenges for the construction of an intelligent information warehouse for urban analysis. In section 6, we offer two further extensions where remote sensing has a major potential role – to ecosystems and to regional (including multi- national and international) systems. There are concluding comments in section 7.

The key overall conclusion is that remote sensing is a potential lead agent in the development of an intelligent information warehouse for urban, regional and ecosystem analysis. The challenge is to integrate remote sensing data with other system data and, above all, the effective connection of remote sensing to modelling that will in turn contribute to enhanced information systems. In the long run, it will contribute to the task of decoding urban 'DNA'.

2. URBAN 'PHYSIOLOGY': THE RETAIL SYSTEM AS AN EXAMPLE.

'Physiology' is concerned with the regular workings of the system of interest. It can be illustrated in the urban case with the retail system. The well-known model provides an interesting archetype on which it is possible to build and to explore new ideas. In this case, we are given a 'structure' of population by place of residence, and their spending power; and the spatial distribution of retail centres with some measure of attractiveness, which for illustrative purposes we can take as related to size – reflecting range of choice and lower prices through scale economies. We can formalize this as follows. Assume a system of discrete zones, labeled as i or j. Let S_{ij} be the flow of retail spend from residents of i to shops in j; let e_i be spending per head and P_i the population of i. W_j is a measure of the attractiveness of shops in j which, for these illustrative purposes, we take as the logarithm of 'size'. The vector $\{W_j\}$ can then be taken as a representation of urban structure – the configuration of W_js. If many W_js are non-zero, then this represents a dispersed system. At the other extreme, if only one is non-zero, then that is a very centralised system. There is clearly, potentially, a measure of order in this specification of structure. The obvious order parameter would be $N(W_j>0)$ – the number of centres which are non-zero. In a fully dispersed system, then $N(W_j>0)$ would be equal to the number of possible centres and would be large; while in a very centralised system, $N(W_j>0)$ would be 1. It is sometimes better to take $N(W_j>M)$ for some constant M greater than 0 as a better measure of structural change.

A spatial interaction model can be built by maximizing an entropy function in the usual way (Wilson, 1967, 1970) to give:

$$S_{ij} = A_i e_i P_i W_j^{\alpha} \exp(-\beta c_{ij}) \quad (1)$$

where

$$A_i = 1/ \Sigma_k W_k^{\alpha} \exp(-\beta c_{ik}) \quad (2)$$

to ensure that

$$\Sigma_j S_{ij} = e_i P_i \quad (3)$$

and

$$\Sigma_{ij} S_{ij} \log W_j = X \quad (4)$$

where $\log W_j$, as we noted earlier, is taken as the measure of consumer benefits and X an estimate of the total benefits achieved.

We also have, implicitly, a constraint on total travel costs:

$$\Sigma_{ij} S_{ij} c_{ij} = C \quad (5)$$

α and β are parameters - actually, the Lagrangian multipliers associated with equations (4) and (5). Note that W_j^{α} can be written

$$W_j^{\alpha} = \exp(\alpha \log W_j) \quad (6)$$

and the core equations can be written

$$S_{ij} = A_i e_i P_i \exp(\alpha \log W_j - \beta c_{ij}) \quad (7)$$

where

$$A_i = 1/ \Sigma_k \exp(\alpha \log W_k - \beta c_{ik}) \quad (8)$$

This shows that $\alpha \log W_k$ can be taken as a measure of benefit to be set against the travel impedance $-\beta c_{ik}$.

Because the matrix is only constrained at the origin end, we can calculate the total flows into destinations as

$$D_j = \Sigma_i S_{ij} = \Sigma_i e_i P_i W_j^{\alpha} \exp(-\beta c_{ij}) / \Sigma_k W_k^{\alpha} \exp(-\beta c_{ik}) \quad (9)$$

Disaggregated versions of these models have been extensively used by retailers to optimise their networks, and so the model is well tested.

3. URBAN STRUCTURE AND THE UNDERPINNING DNA.

We noted in the introduction that urban systems are nonlinear dynamical systems and that we can expect them to exhibit the general characteristics of these systems and, in particular, multiple equilibrium solutions and path dependence. The interaction model of the previous section is an example of 'fast dynamics': the response to any change is sufficiently rapid that the assumption that the system moves rapidly to a new equilibrium is a reasonable one. In the case of structural change, this is not a reasonable assumption, and change is an example of a 'slow dynamics' model. We begin by illustrating these ideas with the retail model and then extend them to the system as a whole.

A suitable hypothesis for representing the dynamics is (Harris and Wilson, 1978):

$$dW_j/dt = \varepsilon (D_j - KW_j) W_j \quad (10)$$

where K is a constant such that KW_j can be taken as the (notional) cost of running the shopping centre in j. This equation then says that if the centre if profitable, it grows; if not, it declines. The parameter ε determines the speed of response to these signals. Again, this is illustrative. It is straightforward to incorporate alternative hypotheses, for example on retail centre rent and cost, but this does not change the essence of the argument.

The equilibrium position is given by

$$D_j = KW_j \quad (11)$$

which can be written out in full [using Dj from equation (9) and hence linking to the physiology] as

$$\Sigma i \{ e_i P_i W_j^{\alpha} \exp(-\beta c_{ij}) / \Sigma_k W_k^{\alpha} \exp(-\beta c_{ik}) \} = KW_j \quad (12)$$

and these are clearly nonlinear simultaneous equations in the $\{W_j\}$.

It is possible to characterise the kinds of configurations that can arise for different regions of α and β space: for larger α and lower β, there are a smaller number of larger centres; and vice versa - as characterized in broad terms by the order parameter, $N(W_j>0)$, defined earlier (cf. Wilson, 1981, Clarke and Wilson, 1985, Clarke, Clarke and Wilson, 1986, Clarke and Wilson, 1986, Lombardo, 1986). This can be interpreted to an extent for a particular zone, say j, by fixing all the W_k, k not equal to j. There are many procedures for solving the equations (12) iteratively and any procedure illustrates the sensitivity to the initial conditions – the path dependence – which is in turn based on the possibility of multiple solutions. [See Wilson (2008-B) for a broad review and how this methodology is more widely applicable.]

The zonal interpretation is shown in Figure 1 (and cf. Wilson, 1981). The left and right hand sides of equation (11) are plotted separately and of course, the intersections are the possible equilibrium points. If $\alpha \leq 1$, there is always a possible equilibrium point, but if $\alpha > 1$, there are three possible cases: only zero as an equilibrium; one additional non-zero stable state; and the limiting case that joins the two. The β value also determines the position of the equilibria. This analysis shows a number of properties that are typical of nonlinear dynamical systems: multiple (system) equilibria and strong path dependence. It also shows that as the parameters α and β (and indeed any other exogenous variables) change slowly, there is the possibility of a sudden change in a zone's state – from development being possible to development not being possible, or vice versa. These kinds of change can be characterised as phase transitions – in this case at a zonal level, but clearly there will be system wide changes of this kind as well. It will turn out that there is the essence of a very powerful tool here for identifying complex phase transitions.

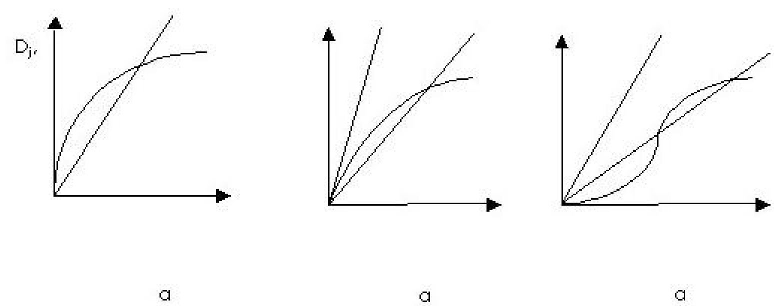

Figure 1. (a) (b) (c)

What we know from the analysis of Figure 1 is that at a zonal level, there are critical values of α and β, for example, beyond which only $W_j = 0$ is a stable solution for that zone. So we know that there are critical points at a zonal level at which, for example, there can be a jump from a finite W_j to a zero W_j (or vice versa) - see Dearden and Wilson, 2008, to illustrate this.) This implies there is a set of α and β at which there will be critical changes somewhere in the system.

In fact, almost any change in model exogenous variables can in principle bring about a phase transition: for example, any change in the $\{e_iP_i\}$. We could possibly take the argument a stage further and build on the fact that equilibrium solutions in nonlinear models are path dependent: we would expect to find phase transitions along some paths but not on others for the same model with different initial conditions. Recall that this analysis is dependent, for a particular W_j, on the set $\{W_k\}$, $k \neq j$, being constant (Wilson, 1988). It is almost certainly a good enough approximation to offer insight, but it is still necessary, as noted earlier, to address the problem of simultaneous variation. The system problem is to predict equilibrium values for the whole set $\{W_j\}$ and the trajectories through time, recognising the points at which phase changes take place.

If we now put time labels on the variables, the path dependence of the system can be expressed by asserting that $\{W_j(t+1)\}$ is dependent on $\{W_j(t)\}$. It is also dependent of course on any changes in $\{e_i(t)\}$, $\{P_i(t)\}$ or $\{c_{ij}(t)\}$ which, from the perspective of the retail model, will be specified exogenously. If we now designate these arrays as $\{W(t), e(t), P(t), c(t)\}$ for brevity, then the composite array $\{W(t), e(t), P(t), c(t)\}$ determines the possible development of the system into the next time period, and in this sense can be thought of as the 'DNA' of the retail system. This 'DNA' will be modified in successive time periods, representing the 'evolution' of the system.

What is particularly interesting is that the evolutionary path may or may not be desirable and this introduces a new planning idea: can we modify the 'DNA' to move the system to a more desirable path? This would be the planning equivalent of genetic medicine: 'genetic planning'.

4. A REPRESENTATION OF A COMPREHENSIVE URBAN SYSTEM

The next step is to extend the argument to the whole urban system. This is done systematically in Wilson (2007). Here, however, we illustrate the argument by defining the additional variables which articulat6e the modelling task as follows:

P_i^k: the number of type k people in zone i
H_i^k: the number of type k houses in zone i
h_i^k: house prices
E_j^{Rk}: the number of type k retail sector jobs in j
E_j^{NRk}: ditto for non-retail
O_i^R: total retail expenditure in i
$D_j^R(t)$: total retail revenue attracted to j
w_j^k: the average wage of a type k job in j
The main interaction variables are[1]:

Y_{ij}^{mVn}, Y_{ij}^{mWn}, Y_{ij}^{mQn}, the flows to work in sectors V, W and Q from population group m in zone i to sector n in zone j for V, W and Q respectively;

N_{ij}^{mnk}, the allocation of type m people who work in sector n in j to type k houses in i;

U_{ij}^{mn}, the flow of type m people in zone i to consumer services of type n in j;

[1] We note the possibility of indices such as m and n themselves being lists

S_{ij}^{mn}, the flow to retail facilities of type n;

J_{ij}^{mn}, the flow of goods from sector m in i to the consumer services sectors n in j;

K_{ij}^{mn}, the flow of goods from sector m in i to the retail services sectors n in j;

M_{ij}^{mn}, the flow of goods from sector m in i to sector n in j.
and these can be represented on a system diagram as in Figure 2.

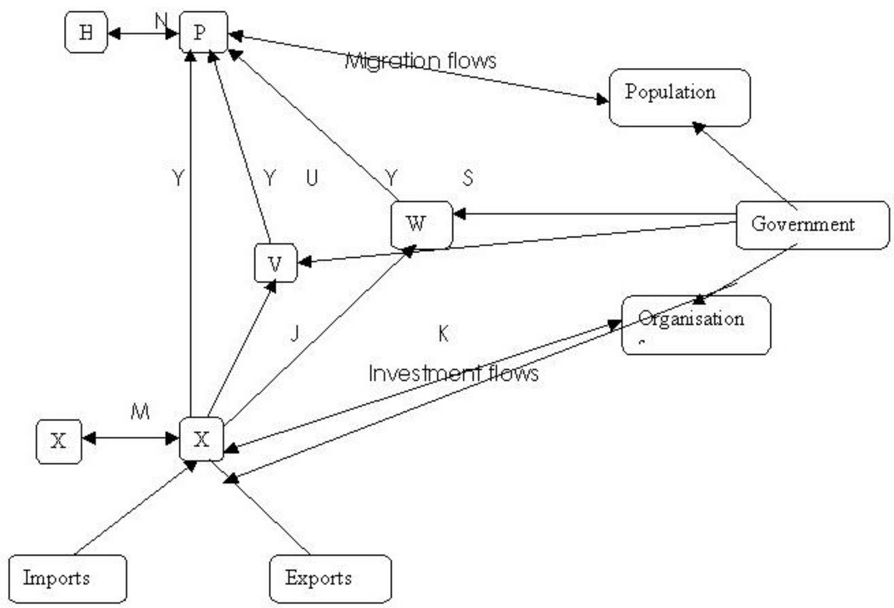

Figure 2

Using an obvious notation, {P, H, V, W, X, L. p, c, G} can then be seen as representing the 'DNA' of the urban system. Can we then address the following kind of question? How does {P, H, V, W, X, L. p, c, G} limit the future development of the city? Technically, this is a question of the regions of phase space which are accessible on the basis of the structure at time zero and any exogenous adjustments that areas made.

Can we summarise this complexity by identifying 'genes'? Candidate genes might include:

Proportions of the population by age group, skill levels, occupations and income

Housing, by price and quality[2]
Access to consumer services
Access to retail
Economic indicators:
'Variety'
Clustering
Proportions in different levels: resources-agriculture-manufacturing-service-post service
Imports and exports; balance of payments
Average income from employment
Income from other sources
Transport connectivity – measured with 'generalised costs'
Land use densities

As indicated earlier – through Wilson (2007) and numerous earlier references – there are comprehensive models that use the kinds of arrays of variables sketched here. What is new here, is the additional analysis – that generalises the retail example of section 3 – that these models can be put into a dynamic framework within which we can characterise the parameters – the 'DNA' – that determine the paths of evolution from each point in time. It can be argued that the rich variable arrays, the associated the model system and the 'DNA' interpretation of system evolution provide the intellectual basis for the core of urban analysis that Longley (2003) was seeking. Indeed, the 'DNA' provides the basis for a new kind of urban classification system – and indeed a classification system for small areas within the city – as the concept scales – upwards and downwards.

5. A REMOTE SENSING-BASED INTELLIGENT INFORMATION WAREHOUSE FOR URBAN ANALYSIS.

We now need to draw together the threads of the argument and demonstrate how remote sensing could contribute to the task of understanding the evolution of urban structure. We consider in turn:

- the integration of remote sensing data with other data and the possibility of using remote sensing to provide estimates of time series of urban development; models can help with this integration; this then provides the data sets for the urban equivalent of X-ray crystallography.
- the path dependence of urban development and the possibility of using a model system to articulate the underlying 'DNA' and the drives towards an equilibrium at different points in time – bearing in mind that the equilibrium is unlikely to be reached;
- coupling the model system to the time evolution of the data, (a) to provide an

[2] 'Quality of environment' to include crime levels etc

understanding of recent history, and (b) to explore the possible 'nearness' of future phase transitions – so that action can be taken if necessary.

The essence of the modelling framework referred to in this paper is that it is rooted in sets of *accounts*. These function as constraints in the development of entropy maximising spatial interaction models and in that context, they can be considered to contain much of our 'knowledge' of the system. In this new context, linking with remote sensing, it is useful to emphasise the degree to which they build consistency into the models: everything has to add up, to be accounted for, in a way that maintains consistency. This, at times, provides a powerful method for estimating missing data. What we ought then to be able to do with remote sensing data is to add it as additional knowledge of the system, possibly as constraints, but also to take advantage of the fact that it is updated much more frequently than, say, a decennial census. We have assumed a discrete zone system. For the sake of argument, assume that as the foundation of an IIW, this is made up of small zones. Remote sensing data for each zone can then be linked to other data so that it becomes possible, for example, to calibrate a model of the relationship between remote sensing observations and small zone population data to enable the estimation of residential density data from remote sensing data alone. This in turn would allow the estimation of small zone population data for time periods not covered by conventional surveys. The same argument could be developed for other kinds of activity. There are also some potential bonuses. Remote sensing would capture much of the transport network – something which is relatively neglected in urban modelling (though obviously not in transport modelling); and also flows on these networks. It would be an interesting subsidiary model challenge to see if this data could be transformed into estimated of flow data – which would involve matching network link flow counts with origin-destination counts. This in turn could connect these analyses – and remote sensing in particular - to the burgeoning – indeed enormous – literature on network dynamics (see Wilson, 2008-B, for as full account of these possibilities).

In this way, the integration of data sources, aided by model predictions of missing data – building on RS data – could provide a detailed account of urban development. These integrated data sets then become the urban equivalents of X-ray photographs of molecules from which structures can be inferred. Model-based analysis could then provide an understanding at each time of the nature of equilibria – the direction of travel of the system towards equilibria. In effect, this would be a charting of the path dependence of urban development, highlighting any significant episodes that represented phase changes.

In particular, this kind of analysis could provide warnings of upcoming possible undesirable phase transitions; or could be the basis for the recommendation of planning actions to bring about phase transitions that would lead to desirable states – the 'genetic planning' argument. A remote sensing-driven model-based IIW could provide planners with a description of land packets that was available for transformation and different kinds of development for further model-based analysis of future development scenarios. The beginnings of this kind of system have been explored by Dearden and Wilson (2008) and

Figure 3 is taken from that paper, showing how timelines involving phase changes can be modelled and displayed.

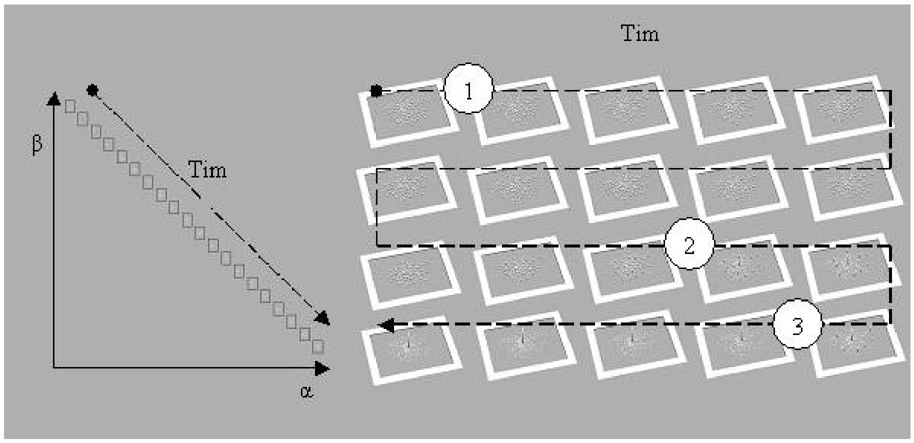

Figure3. An hypothetical time line – from Dearden and Wilson (2008)

How would this work in practice? Key drivers include migration, investment by organisations and investment by Government agencies (local, regional, national or international) – the possible interventions already anticipated in earlier sections through the introduction of the G-arrays. The genes then come into play by considering the drivers of migration, economic investment and Government investment. Migration might be driven by housing and work availability and the quality of consumer and retail services and the quality of the social environment; economic investment by profitability which in turn will depend on the availability of skilled labour, other inputs and markets. Both will be influenced by transport connectivities. The various submodels will have to incorporate some preferences that are not economic – for example, for (relative) social differentiation in residential areas (and correspondingly in schools). Government agencies will be obliged to supply what are agreed to be public goods – such as schools and hospitals – and to make these sufficiently attractive to lever migration and economic investment in appropriate directions. In particular, in terms of the model-based foundations, government agencies need to find the bifurcation points at which sudden changes for the better can be achieved.

An analysis of the physiology then reveals a high degree of interdependence. (An interesting question is whether this can be measured in some way?) Much of the interdependence is rooted in the input-output model. What we have seen already is that the set of drivers are spread around that model: it is no longer something simple like final demand that is determining. It is this insight, and the known path dependence of dynamical systems, that makes the notion of a path through a sequence of ‘initial conditions' an important idea.

At the starting point, households will be located, workers will be employed, deriving income which is channelled back to households. Organisations will be profitable, or not; or funded by the Government, or not. What the different interaction models represent is an intersection of supply and demand but which, in each case, is mediated by other models. Ideally, the interdependence needs to be made explicit through linking submodels. For example, consider household income derived from employment. Ideally we need a model which shows how that is allocated between different areas of expenditure and what ends up in, say, the retailing 'pot' is then part of a demand function for retail goods. But that cannot be separated from earned income and the way it is divided. Typically, even comprehensive, urban models do not tackle this issue.

Assume then that the population and organisations (again internal and external in each case) and government agencies each respond to signals – indicators – that give them measures of the current situation. Such indicators can be calculated from the model outputs and, in effect, they are the kinds of things that we have already recorded in the core model equations (in Wilson, 2008-B) for the slow dynamics: retailers responding to profits and losses for example. The particularly interesting case is that of government agencies as noted above. These are then the mechanisms for representing the evolution of the structural variables.

6. Extensions to ecosystems and regional systems.

It is appropriate to add two areas that relate to the wider applicability of the model system which are particularly relevant to the use of remote sensing data. First, it has been shown that the same modelling principles apply to ecosystems (Wilson, 2006) and since remote sensing data is very rich in this sphere – indeed can capture a much greater percentage of the necessary data than is the case with urban systems - there ought to be fruitful applications. Secondly, we should note that we have focused here, for illustrative purposes, on cities. It can be argued that in order to effectively represent urban *and regional* dynamics in a comprehensive way, it is necessary to develop a linked hierarchy of models (Wilson, 2008-A). There are potentially four layers:

(1) nations within the global system;
(2) regions within a country;
(3) cities within a region;
(4) intra-urban structure.

and we have concentrated only on the fourth. The top three layers involve different styles of modelling – demographic and economic input-output models – but still with spatial interaction components made up of migration and trade flows. It would be particularly exciting to develop a GIS/IIW at the top – global – scale that relied on satellite-based remote sensing to provide the underpinnings of the wider picture.

7. CONCLUDING COMMENTS

We have indicated that urban and regional modelling has now matured as a field to the point where it begins to be possible to understand evolution, path dependence and the nature of phase transitions. What could hold back further, and especially empirical, development is the lack of adequate time series data. Remote sensing data could be used to meet this challenge through the development of methods to integrate its time series with other data sources – within a modelling framework, because that framework in turn can be used to aid data estimation. If this can be achieved, through what was described above as a remote sensing driven model-based intelligent information warehouse, then this would provide the foundations for a substantial scientific advance.

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