

Quiz 3, Stat3

Time allowed 45 mins

Total marks 15

1. (3) Classify the following variables for a BSDS student as ordinal, nominal and continuous.
 - (a) Location (Bangalore/Delhi/Kolkata)
 - (b) Frequency of physical exercise (Never/Rarely/Often/Regularly)
 - (c) Percentage marks in first year
2. (X_1, X_2, X_3) is jointly distributed as $Multinomial_3(n, (p_1, p_2, p_3))$, where $p_1 + p_2 + p_3 = 1$.
Let $\hat{p}_i = X_i/n$ for $i = 1, 2$. From your probability class you should know that the marginal distribution of X_i is $\text{Bin}(n, p_i)$, $i=1,2$. (no need to show this.)
 - (a) (2) Find the expectation and variance of $\hat{p}_i$, $i=1,2$.
 - (b) (2) Write down the joint distribution of (X_1, X_2) .
 - (c) (3) Using this show that the covariance of $(\hat{p}_1, \hat{p}_2)$ is $-p_1 p_2 / n$.
 - (d) (2) Using the previous two parts, find the variance of $(\hat{p}_1 - \hat{p}_2) = \sigma^2$ (say).
 - (e) (3) It can be shown that $(\hat{p}_1 - p_1 - \hat{p}_2 + p_2)/\sigma$ converges to $\mathcal{N}(0, 1)$ in distribution as n goes to infinity (you don't need to prove this.)
Use this to construct a hypothesis test for testing the null hypothesis $p_i = 1/3, i = 1, 2, 3$ at level α . Give the test statistic and the rejection region in terms of the standard normal cdf.