

Quiz 2, Stat3

Time allowed 45 mins

Consider simple linear regression and the problem of testing $\beta_1 = 0$. We can do this in two ways.

- A Use a t -test based on $\hat{\beta}_1 \sim \mathcal{N}(\beta_1, \sigma^2/S_{XX})$
- B Use the F test in the ANOVA

Answer the following questions in this context.

1. Write down an estimator for σ^2 that is independent of $\hat{\beta}_1$.
2. What is the distribution of the estimator in part (1)?
3. Write down the t-statistic mentioned in (A) and its distribution.
4. Show that the F-statistic in (B) is

$$\frac{S_{YY} - \sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \hat{y}_i)^2 / (n - 2)}$$

5. Show that the numerator in the F-statistic equals S_{XY}^2/S_{XX} .
6. Hence show that the F statistic in (B) is square of the t-statistic in (A).
7. Conclude that the two tests (A) and (B) are equivalent by expressing the t and F distributions in terms of independent normals.