Homework 4

1. The tensile strength of Portland cement is being studied. Four different mixing techniques can be used economically. A completely randomized experiment was conducted and the followng data was collected.

Technique	Tensile strength (lb/in^2)			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

   (a) Test the hypothesis that mixing techniques affect the tensile strength of the cement. Use $\alpha = 0.05$.

   (b) Construct a normal probability plot of residuals. What conclusion can be drawn?

   (c) Plot the residuals vs the fitted values. What conclusion can be drawn?

   (d) Make comparisons between pairs of means without any adjustment for multiple comparions.

   (e) Use Bonferroni method to make comparisons between pairs of means.

   (f) Do the two methods give different results? If so, explain where this difference is coming from.

2. A single factor completely randomized design has six levels of the factor. There are five replicates at each level and the total sum of squares is 900.25. The treatment sum of squares is 750.5.

   (a) What is the estimate of the error variance?

   (b) What proportion of variability is explained by the treatment effect?

   (c) What is the F-statistic and corresponding p-value?

3. Consider an experiment with continuous response (y) and one factor (x) with k levels. Suppose there are n_i independent measurements at each level for $i = 1, \cdots, k$. A linear combination $\sum c_i \beta_i$ of the level means (β_i) is called a contrast if $\sum c_i = 0$. For eg, $\beta_2 - \beta_1$ is a contrast.

   (a) Write down the linear model and explain the problem of non-identifiability.

   (b) If we want to obtain point estimates of the level means, then what should be the procedure for removing non-identifiability?

   (c) Suppose we estimate a contrast by plugging in the point estimates of each level mean ($\sum_{j=1}^{n_i} y_{ij}/n_i$). Find the variance of this estimate.

   (d) Using the above and assuming independent normal errors, construct a test for the hypothesis $H_0 : \sum c_i \beta_i = 0$.