1. (3+3) A sequence of random variables $X_n$ is defined as $X_n$=the number of trials required to obtain the first success when the probability of success in each trial is $1/n$. Let $Y_n = X_n/n$.

    (a) Find $P(Y_n \leq x)$.

    (b) Show that the asymptotic distribution of $Y_n$ is exponential.

2. (2+4 ) Let $X_i, i = 1, 2, \cdots, n$ be independent Bernoulli($p$) random variables and let $T_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. A popular parameter is the odds $\frac{p}{1-p}$. For example, if the data represent the outcomes of a medical treatment with $p = 2/3$, then a person has odds 2:1 of getting better. We consider the estimate $Y_n = \frac{T_n}{1-T_n}$ for the parameter $h(p) = \frac{p}{1-p}$.

    (a) What is the limiting distribution of $\sqrt{n}(T_n - p)$ as $n \to \infty$.

    (b) Find the limiting distribution of $\sqrt{n}(Y_n - h(p))$ as $n \to \infty$ using the Delta method.

3. (4+2+2) Let $X_i, i = 1, 2, \cdots, n$ be a random sample of size $n$ from $f_\theta(x) = (1-\theta)\theta^x; x = 0, 1, 2, \cdots; \theta \in (0; 1)$.

    (a) Find the MLE of $\theta$ and show that it is consistent.

    (b) Find the Fisher Information.

    (c) Find the asymptotic distribution of the MLE.