Quiz 3
21.02.20

1. (4+2) Let $X_1, \cdots, X_n$ be iid Geometric(p) random variables, each X_i denoting the number of Bernoulli(p) trials required for the first success. Show that $\bar{X}$ attains the Cramer-Rao lower bound. Hence conclude that $\bar{X}$ is UMVUE for $1/p$.

2. (2+2+2+2) Suppose $\delta(X)$ is a Bayes estimator, that is, the posterior mean of parameter $g(\theta)$ under prior π. Suppose $\delta(X)$ is also unbiased for $g(\theta)$.
   (a) Show, by conditioning on X, that $E(\delta(X)g(\theta)) = E(\delta^2(X))$.
   (b) Show, by conditioning on θ, that $E(\delta(X)g(\theta)) = E(g^2(\theta))$.
   (c) Conclude that $E(\delta(X) - g(\theta))^2 = 0$.
   (d) If $X_1, \cdots, X_n$ are iid $\mathcal{N}(\mu, 1)$, then use the above result to conclude that $\bar{X}$ cannot be a Bayes estimator for μ under any prior.

3. (2+4) Let $X_1, \cdots, X_n$ be iid observations from population with mean μ and variance σ^2.
   (a) Show that the estimator $\sum_{i=1}^n a_i X_i$ is unbiased for μ iff $\sum_{i=1}^n a_i = 1$
   (b) Among all estimators of the above form find the one with minimum variance.