Quiz 2
07.02.20

For problems 1-3, assume that $(X_1, \cdots, X_n) \sim i.i.d.Unif([0, \theta])$.
Note: $E(X) = \theta/2, E(X_{(k)}) = k\theta/(n+1)$.

1. (2+4) Find a one-dimensional sufficient statistic T for θ. Find an estimating equation for θ that depends on $(X_1, \cdots, X_n)$ only through T.

2. (4) Find the MLE of $\theta/2$.

3. (3+3) Show that the Pareto prior, given below as π, is a conjugate distribution and find the posterior mean under Pareto(α, β) prior.

$$\pi(\theta|\alpha, \beta) = \frac{\alpha\beta^\alpha}{\theta^{\alpha+1}}, \theta \geq \beta > 0$$

Note: $E(\theta|\alpha, \beta) = \frac{\alpha\beta}{\alpha-1}$ for the above distribution.

4. (4) Suppose $X \sim Bin(n, 1/3)$ where n is the unknown parameter of interest. Find the MLE of n when $X=5$.