Statistics II, Quiz 1, 24.01.20

1. (3) Suppose $Y_i = \sum_{j=1}^{p} Z_{ij}\beta_j + \epsilon_i, \quad \text{where} \quad \epsilon_i \sim \mathcal{N}(0, \sigma^2)$ and independent for $1 \leq i \leq n$ and Z_{ij} are fixed and known. If $n < p$, then show that $\beta = (\beta_1, ..., \beta_p)$ is not identifiable.

2. (5) $X_1, \cdots, X_n$ are iid Exponential(λ). Show that $E(X_1 / \sum_{i=1}^{n} X_i) = 1/n$.

3. (10) Let $f(\mathbf{x}|\theta)$ be the pdf or pmf of $\mathbf{X}$. Suppose there exists a statistic $T(\mathbf{X})$ such that, for any two points $\mathbf{x}$ and $\mathbf{y}$, the ratio $f(\mathbf{x}|\theta)/f(\mathbf{y}|\theta)$ is constant as a function of θ iff $T(\mathbf{x}) = T(\mathbf{y})$. Then prove that $T(\mathbf{X})$ is a minimal sufficient statistic.

4. (2) The distribution of X belongs to a one-dimensional exponential family and we have iid observations $X_1, \cdots X_n$ from this distribution. What is the dimension of the natural sufficient statistic for the joint distribution of $(X_1, \cdots, X_n)$?