Modeling the Reliability of Ball Bearings: A Statistical Analysis

V. Sai Prabhav
G. Sai Shashank
A. Revanth Sairam

Indian Statistical Institute, Bangalore Centre
9th April, 2025

Abstract

This report presents a comprehensive statistical analysis of ball bearing reliability data originally published by Lieblein and Zelen (1956). Using multiple linear regression techniques, we examine the relationship between bearing life and key engineering parameters. The analysis focuses on testing hypotheses about parameter equality across different manufacturers and bearing types, with particular attention to validating the industry-standard exponent value of 3 in the fatigue-life equation. Our findings support the traditional model while revealing interesting variations between manufacturers. The report includes detailed statistical methodology, graphical representations of the data, and interpretation of results.

Contents

1 Introduction **2**
1.1 Background 2
1.2 Study Objectives 2

2 Data Description **3**
2.1 Data Source 3
2.2 Key Variables 3

3 Theoretical Framework **4**
3.1 Generalized Fatigue-Life Model 4
3.2 Log-Linear Transformation 4

4 Exploratory Data Analysis **5**
4.1 Distribution of Key Variables 5
4.2 Relationships Between Variables 6

5 Methodology **9**
5.1 Hypotheses 9
5.2 F-Test Framework 9

6 Results **10**
6.1 Hypothesis (a): Parameter Equality Across Companies 10
6.2 Hypothesis (b): Common β_3 Across Companies 10
6.3 Hypothesis (c): Parameter Equality Across Types in Company B 11
6.4 Hypothesis (d): Common β_3 Across Types in Company B 11
6.5 Hypothesis (e): $p = 3$ Across Types in Company B 11

7 Discussion **13**
7.1 Interpretation of Results 13
7.2 Comparison with Previous Studies 13
7.3 Limitations 13

8 Conclusion **14**

Introduction

1.1 Background

Ball bearings are critical components in mechanical systems, and their reliability directly impacts the performance and longevity of machinery. The fatigue life of ball bearings has been studied extensively, with the ISO Standard 281 providing the fundamental relationship between bearing life and operational parameters.
The standard model relates the rating life L_{10} (the number of revolutions at which 90% of bearings survive) to the applied load P through the equation:

$$L_{10} = \left(\frac{C}{P}\right)^p$$

where C is the basic dynamic load rating and p is an exponent traditionally assumed to be 3 for ball bearings.

1.2 Study Objectives

This investigation has several key objectives:

- To validate the theoretical model using real-world test data
- To examine whether the exponent $p = 3$ holds across different manufacturers and bearing types
- To determine if manufacturers use significantly different parameter values in their bearing designs
- To provide a comprehensive statistical analysis of bearing reliability data

Data Description

2.1 Data Source

The data comes from endurance tests conducted by three major bearing manufacturers (anonymized as Companies A, B, and C) and originally published by Lieblein and Zelen (1956). The dataset contains 210 test records, each representing a group of bearings tested under identical conditions.

2.2 Key Variables

The dataset includes the following key variables for each test:

- Company identifier (1, 2, or 3)
- Test number and year
- Number of bearings in the test
- Load (P) in pounds
- Number of balls (Z)
- Ball diameter (D) in inches
- L_{10} and L_{50} life estimates (in millions of revolutions)
- Weibull slope (not used in our analysis)
- Bearing type (for Company B only)

Table 2.1: Summary of Ball-Bearing Data

Company	**# of Test Groups**	**# of Bearings**
A	50	1,259
B	148	3,289
B-1	37	—
B-2	94	—
B-3	17	—
C	12	291

Theoretical Framework

3.1 Generalized Fatigue-Life Model

The generalized model from Lundberg and Palmgren (1947) relates bearing life to its characteristics:

$$L = \left(\frac{f Z^a D^b}{P} \right)^p$$

where:

- L: Bearing life at a given percentile
- P: Applied dynamic load
- Z: Number of balls
- D: Ball diameter
- f: Proportionality constant
- a, b: Empirically determined exponents
- p: Fatigue-life exponent

3.2 Log-Linear Transformation

For statistical analysis, we transform the model by taking natural logarithms:

$$\ln(L) = \alpha + \beta_1 \ln(Z) + \beta_2 \ln(D) + \beta_3 \ln(P)$$

where:

$$\begin{aligned} \alpha &= p \ln(f) \\ \beta_1 &= ap \\ \beta_2 &= bp \\ \beta_3 &= -p \end{aligned}$$

This linear form allows us to use multiple regression techniques to estimate the parameters and test hypotheses about their values.

Exploratory Data Analysis

4.1 Distribution of Key Variables

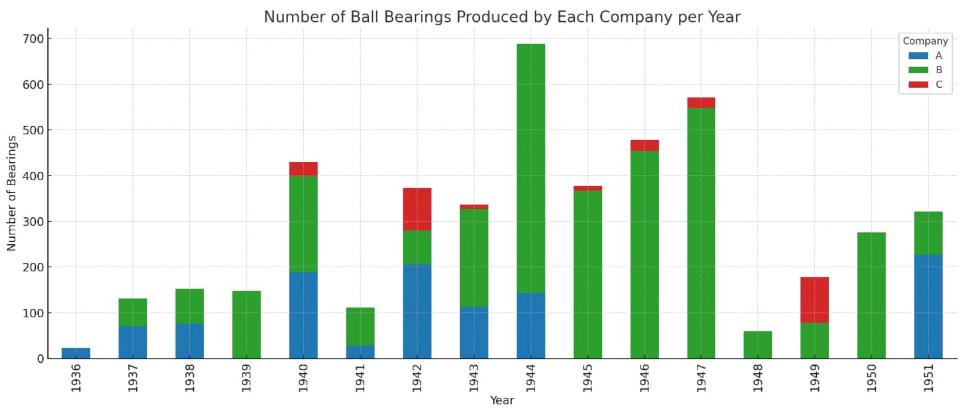

Figure 4.1: Distribution of test years across companies

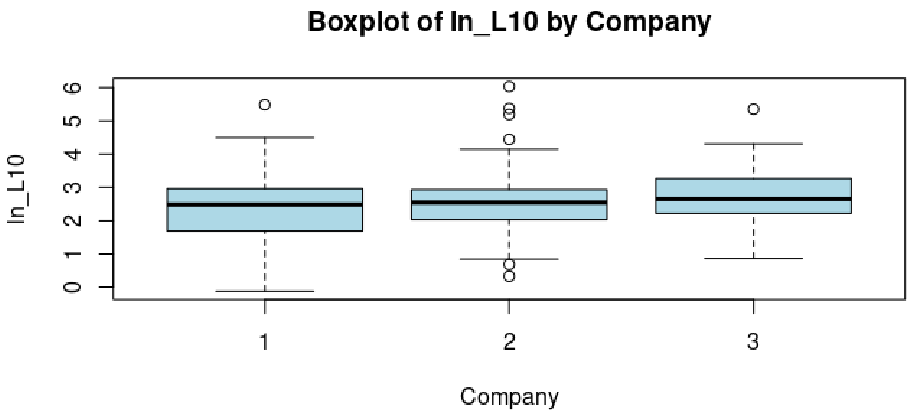

Figure 4.2: Boxplot of L_{10} values by company

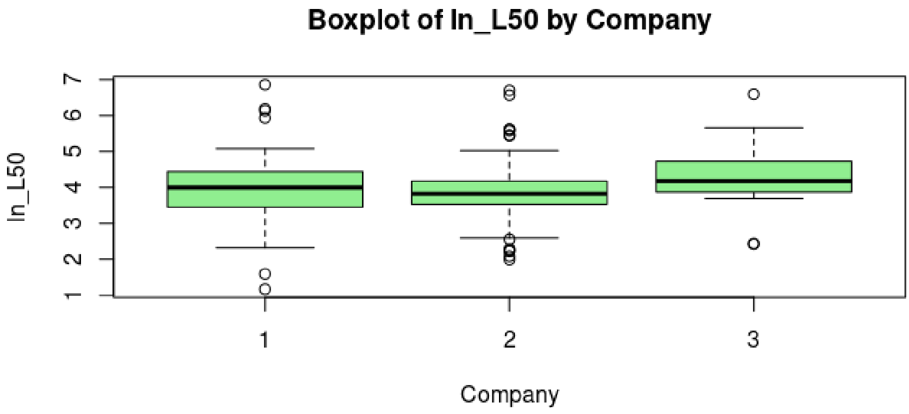

Figure 4.3: Boxplot of L_{50} values by company

4.2 Relationships Between Variables

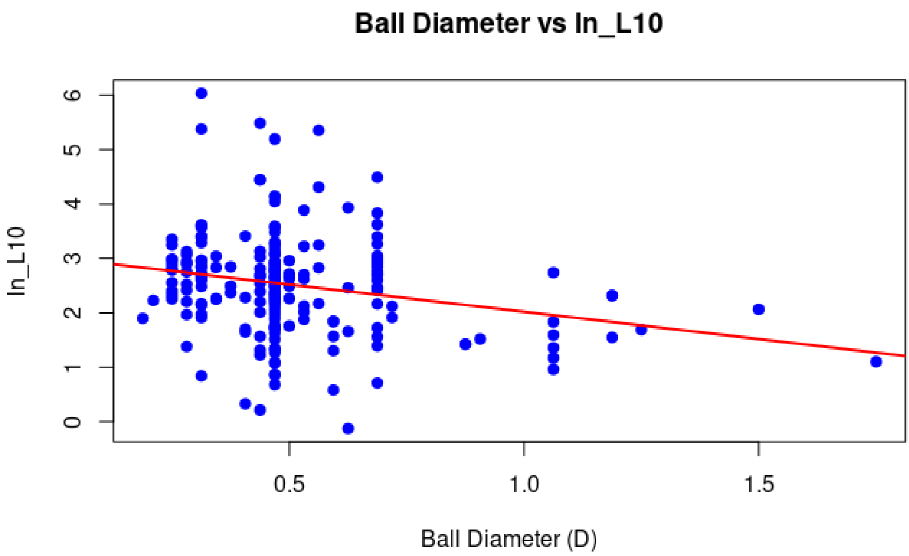

Figure 4.4: Relationship between ball diameter (D) and L_{10}

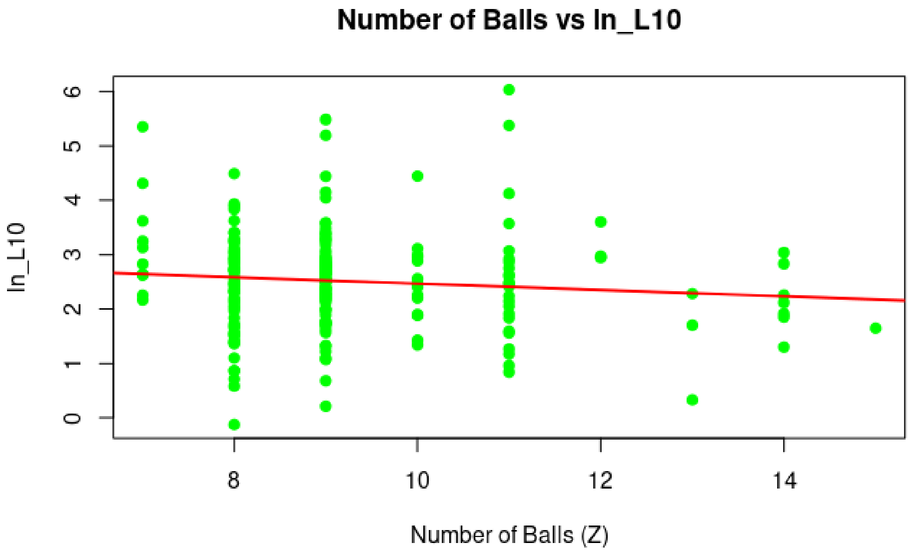

Figure 4.5: Relationship between number of balls (Z) and L_{10}

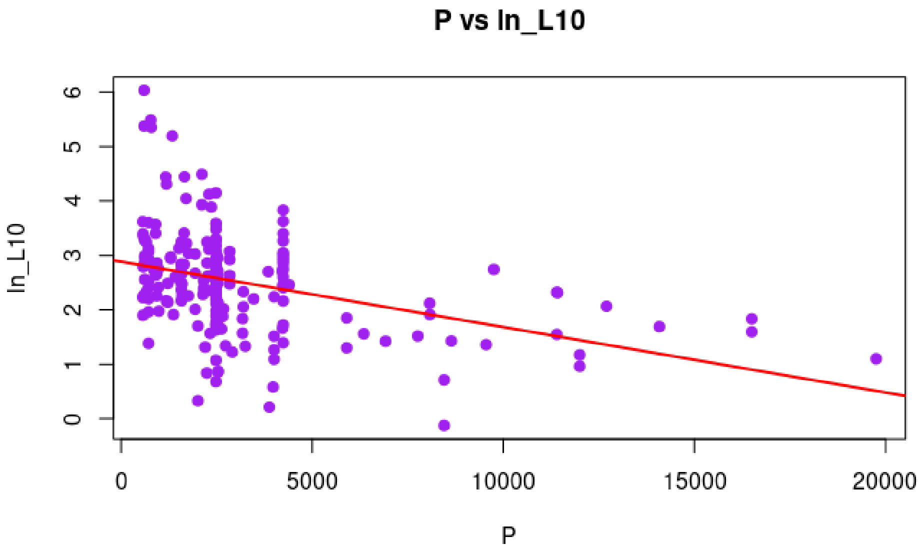

Figure 4.6: Relationship between load (P) and L_{10}

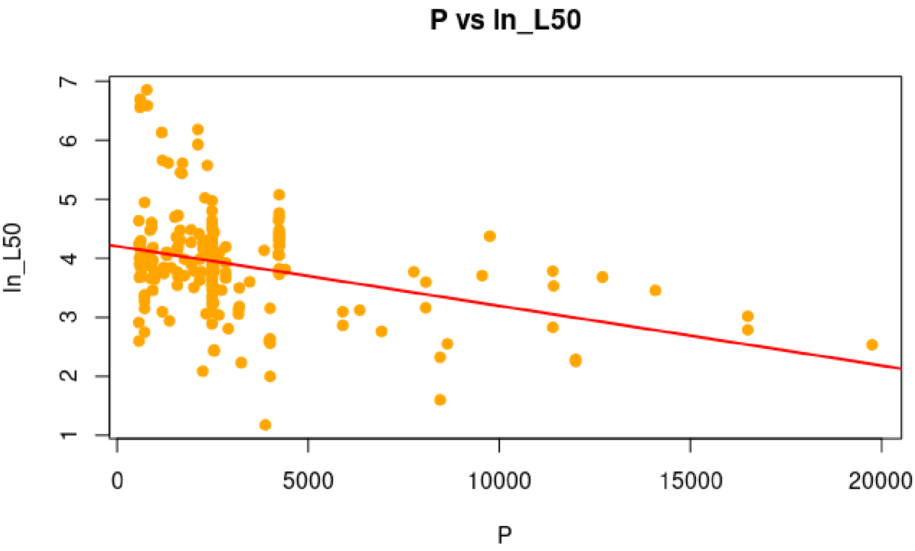

Figure 4.7: Relationship between load (P) and L_{50}

Methodology

5.1 Hypotheses

We test five main hypotheses about the model parameters:

(a) All parameters $(\alpha, \beta_1, \beta_2, \beta_3)$ are identical across companies

(b) The parameter β_3 (and thus p) is identical across companies

(c) All parameters are identical across bearing types within Company B

(d) The parameter β_3 is identical across bearing types within Company B

(e) The parameter β_3 corresponds to $p = 3$ for bearing types within Company B

5.2 F-Test Framework

We use F-tests to compare nested regression models. The F-statistic is calculated as:

$$F = \frac{(RSS_0 - RSS_1)/(p_1 - p_0)}{RSS_1/(n - p_1)}$$

where:

- RSS_0: Residual sum of squares for the simpler model
- RSS_1: Residual sum of squares for the more complex model
- p_0, p_1: Number of parameters in each model
- n: Number of observations

A significant F-statistic (low p-value) indicates that the more complex model provides a better fit to the data.

Results

6.1 Hypothesis (a): Parameter Equality Across Companies

```
Analysis of Variance Table

Model 1: ln_L10 ~ (ln_Z + ln_D + ln_P)
Model 2: ln_L10 ~ Company * (ln_Z + ln_D + ln_P)
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    206 2215.4
2    198 1944.5  8    270.88 3.4478 0.0009674 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 6.1: ANOVA results for Hypothesis (a)

The extremely small p-value (< 0.001) leads us to reject the null hypothesis. We conclude that at least one of the parameters $(\alpha, \beta_1, \beta_2, \beta_3)$ differs between companies.

6.2 Hypothesis (b): Common β_3 Across Companies

```
Analysis of Variance Table

Model 1: ln_L10 ~ Company * (ln_Z + ln_D) + ln_P
Model 2: ln_L10 ~ Company * (ln_Z + ln_D + ln_P)
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    200 1950.1
2    198 1944.5  2    5.5869 0.2844 0.7527
```

Figure 6.2: ANOVA results for Hypothesis (b)

The large p-value (0.753) indicates we cannot reject the null hypothesis. The parameter β_3 (and thus p) appears to be the same across companies.
The estimated common value of p (from the "common p" regression) is 2.876 (SE = 0.178) for L_{10} and 2.804 (SE = 0.156) for L_{50}. Both estimates include $p = 3$ within their 95% confidence intervals.

6.3 Hypothesis (c): Parameter Equality Across Types in Company B

```
Analysis of Variance Table

Model 1: ln_L10 ~ ln_Z + ln_D + ln_P
Model 2: ln_L10 ~ Bearing_type * (ln_Z + ln_D + ln_P)
  Res.Df    RSS Df Sum of Sq      F  Pr(>F)
1    144 1405.8
2    136 1258.3  8    147.54 1.9934 0.05169 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 6.3: ANOVA results for Hypothesis (c)

The p-value (0.052) is borderline significant. At the 5% level, we might consider rejecting the null hypothesis, suggesting that at least one parameter differs between bearing types in Company B.

6.4 Hypothesis (d): Common β_3 Across Types in Company B

```
Analysis of Variance Table

Model 1: ln_L10 ~ +Bearing_type * (ln_Z + ln_D) + ln_P
Model 2: ln_L10 ~ Bearing_type * (ln_Z + ln_D + ln_P)
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    138 1291.8
2    136 1258.3  2    33.538 1.8125 0.1672
```

Figure 6.4: ANOVA results for Hypothesis (d)

The large p-value (0.167) suggests we cannot reject the null hypothesis. The parameter β_3 appears consistent across bearing types within Company B.

6.5 Hypothesis (e): $p = 3$ Across Types in Company B

```
Analysis of Variance Table

Model 1: ln_L10 ~ +Bearing_type * (ln_Z + ln_D)
Model 2: ln_L10 ~ Bearing_type * (ln_Z + ln_D + ln_P)
  Res.Df    RSS Df Sum of Sq    F Pr(>F)
1    139 1291.8
2    136 1258.3  3    33.585 1.21 0.3086
```

Figure 6.5: ANOVA results for Hypothesis (e)

The p-value (0.309) is not significant, so we cannot reject the hypothesis that $p = 3$ for bearing types in Company B.

Discussion

7.1 Interpretation of Results

Our analysis provides mixed evidence about the uniformity of bearing reliability models across manufacturers and types:

- The significant result for Hypothesis (a) suggests that manufacturers may use different design parameters (α, β_1, β_2) even if they share the same load-life exponent (p).
- The non-significant result for Hypothesis (b) supports the industry practice of using a common p value across manufacturers.
- The borderline result for Hypothesis (c) hints at possible differences in how Company B designs its various bearing types.
- The non-significant results for Hypotheses (d) and (e) support the traditional value of $p = 3$ for ball bearings.

7.2 Comparison with Previous Studies

Our findings generally agree with Lieblein and Zelen's original analysis, confirming that:

- The exponent p is approximately 3 across different bearing types and manufacturers
- The model structure is appropriate for describing bearing fatigue life
- Some parameter differences exist between manufacturers and bearing types

7.3 Limitations

Several limitations should be noted:

- The data are from the 1950s - modern bearing technology may differ
- The number of tests varies greatly between companies
- Some bearing types have very few test groups
- The analysis assumes the Weibull distribution for failure times

Conclusion

This comprehensive analysis of ball bearing reliability data has validated key aspects of the industry-standard model while revealing interesting variations between manufacturers. The traditional exponent value of $p = 3$ receives strong support from the data, though some design parameters appear to vary between manufacturers and bearing types. The study demonstrates the power of statistical methods in engineering applications, particularly multiple regression analysis for testing theoretically derived models. The results provide valuable insights for both bearing manufacturers and users regarding the reliability characteristics of these critical mechanical components.

References

- Caroni, C. (2002). "Modeling the Reliability of Ball Bearings." *Journal of Statistics Education*, 10(3).
- Lieblein, J., and Zelen, M. (1956). "Statistical investigation of the fatigue life of deep-groove ball bearings." *Journal of Research of the National Bureau of Standards*, 57, 273-316.
- Lundberg, G., and Palmgren, A. (1947). "Dynamic capacity of rolling bearings." *Acta Polytechnica, Mechanical Engineering Series*, 1(3).
- Meeker, W. Q., and Escobar, L. A. (1998). *Statistical Methods for Reliability Data*. Wiley.