INDIAN STATISTICAL INSTITUTE

CHENNAI CENTRE

M.STAT I. 2016-17 Semester II

Large Sample Statistical Methods

Mid Semester Examination

Maximum Marks 100

Numbers in brackets indicate points for each question. Total marks is 105.

You can answer as much as you can. However, maximum you can score is 100.

Date: 21 February 2017 Duration: 3 hours

1 $X_1, \cdots, X_n$ are i.i.d. with the Binomial(m, p) distribution, where both m and p are unknown. The Method of Moments(MoM) equates the sample moments of the observations to their expected values.

(a) Write down the first two method of moments equations and solve them to find the estimators $\hat{m}$ and $\hat{p}$. [10]

(b) Show that $\hat{m}$ and $\hat{p}$ are consistent. [10]

(c) Using Cramer Wold device, show that as $n \to \infty$

$$\sqrt{n}\begin{pmatrix} \bar{X} - mp \\ \frac{1}{n}\sum_{i=1}^{n} X_i^2 - mp(1+(m-1)p) \end{pmatrix} \Rightarrow \mathcal{N}_2(0, \Sigma)$$

where Σ is a covariance matrix. Find Σ. [10]

(d) Show that the MoM estimators $\hat{m}$ and $\hat{p}$ are jointly asymptotically normal in the sense that

$$\sqrt{n}\begin{pmatrix} \hat{m} - m \\ \hat{p} - p \end{pmatrix} \Rightarrow \mathcal{N}_2(0, V)$$

where V is another covariance matrix. Find V . [10]

2 Let $X_1, X_2, \cdots$ be a sequence of random variables. Show that $X_n \xrightarrow{P} 0$ as $n \to \infty$ if and only if

$$\mathrm{E}\left(\frac{|X_n|}{1+|X_n|}\right) \to 0 \quad \text{as} \quad n \to \infty$$

[15]

3 Suppose that$X_1, \cdots, X_n$ are i.i.d. observations whose mgf exists for some $t > 0$. Show that $X_{(n)} = O_P(\log(n))$, where $X_{(n)}$ is the largest order-statistic. [15]

4 Let $X_1, \cdots, X_n$ be i.i.d. observations with an unknown continuous distribution F. Suppose that one wishes to determine the sample size n so that the probability is at least 95% that the maximum difference between the empirical d.f. F_n and F is less than 0.1. Show that the result does not depend on F and hence we can use the uniform distribution without loss of generality. [10]

5 Find the MLE and its asymptotic distribution given a random sample of size n from $f_\theta(x) = (1-\theta)\theta^x, x = 0, 1, 2, \cdots, \quad \theta \in (0, 1)$. [10]

6 Suppose $X_1, \cdots, X_n$ is an iid sample with $P(X_i \leq x) = F(x-\theta)$, where $F(x)$ is symmetric about zero. We wish to estimate θ by $(Q_p + Q_{1-p})/2$, where Q_p and Q_{1-p} are the p and $1-p$ sample quantiles, respectively. Find the smallest possible asymptotic variance for the estimator and the p for which it is achieved when F is Standard double exponential. [15]

———*** xXx ***———