

INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT I. 2013-14 Semester I
Large Sample Theory

Each question carries 14 points. Answer as many as you want. Maximum you can score is 100.

Duration: 3 hours

- 1 $Y_i = X_i^2$ where $X_i \sim \mathcal{N}(0, \sigma^2)$ and $T_n = \bar{Y}_n$.
 - (a) Show that T_n is unbiased for σ^2 .
 - (b) Find the variance stabilizing transformation h of T_n .
 - (c) Find the asymptotic distribution of $\sqrt{n}(h(T_n) - h(\sigma^2))$.
- 2 Let X_1, \dots, X_n be iid $N(0, \sigma^2)$. Find the asymptotic relative efficiency of the estimator $\delta_n = \sqrt{\frac{\pi}{2}} \sum |X_i| / n$ with respect to the MLE $= \sqrt{\sum X_i^2 / n}$ of σ
- 3 A sequence X_n of random variables converge in distribution to a random variable X .
 - (a) Construct random variable Y_n and Y such that $Y_n \stackrel{d}{=} X_n$, $Y \stackrel{d}{=} X$ and $Y_n \xrightarrow{wp1} Y$. (No proof required. Just define the random variables.)
 - (b) Show that for a continuous function g , $g(Y_n) \xrightarrow{wp1} g(Y)$
 - (c) Conclude that $g(X_n)$ converge in distribution to $g(X)$
- 4 Give examples of the following and justify your claim in each case
 - (a) A sequence of random variables X_n that converge in distribution to X but not in probability.
 - (b) A sequence of random variables X_n that converge in probability to X but not with probability one.
- 5 Derive the asymptotic distribution of the sample T-statistic. You can state and use any theorem proved in class.
- 6 Use the asymptotic joint distribution of sample quantiles to derive the asymptotic distribution of the interquartile range.
- 7 $F_n \Rightarrow F$ and F is continuous. Show that the convergence is uniform.
- 8 Let $X_n \sim \text{Bin}(n, p)$.
 - (a) Find the asymptotic distribution of $\sqrt{n}(X/n - p)$.
 - (b) Find a consistent estimator of pq .
 - (c) Use the results above and Slutsky's theorem to obtain approximate confidence intervals for p .