INDIAN STATISTICAL INSTITUTE
M.STAT Second Year
2017-18 Semester II

Computational Finance
Midterm Examination

Points for each question is in brackets. Total Points 100.
Students are allowed to bring 2 pages (one-sided) of hand-written notes
Duration: 3 hours

1. (5+15+10) The objective of this problem is to show that Binomial Option pricing formula with
$$u = e^{\sigma\sqrt{\Delta t}}, d = 1/u \quad \text{and} \quad p = \frac{e^{r\Delta t} - d}{u - d}$$
approximately solves the BS equation as $\Delta t \to 0$ and to find the Binomial option price numerically.
   (a) Write down the relationship induced by no arbitrage, between prices of an option at time t and $t + \Delta t$, in the single period Binomial model.
   (b) Use Taylor expansion and take limit at $\Delta t \to 0$ to show that the relationship in (a) leads to the Black Scholes PDE.
   (c) Write a program to implement the Binomial model above to price a CALL option. The inputs are the asset price S_0, the strike price K, the riskless interest rate r, the current time t and the maturity time T, the volatility σ, and the number of steps n.
2. (10) Show that for estimating $E[f(Z)]$ based on an antithetic pair $(Z, -Z)$, where $Z \sim \mathcal{N}(0, I)$, antithetic sampling eliminates all variance if f is antisymmetric.
3. (10) Describe the Mersenne twister random number generator algorithm.
4. (15) Obtain the price of a geometric average PUT option analytically when the underlying STOCK follows geometric Brownian motion.
5. (10+10) Let f be a twice continuously differentiable function on $[a, b]$. Suppose $f(x) = P_1(x) + E(x)$, where P_1 is the polynomial of degree one in the trapezoid rule. Show that $\mid E(x) \mid \leq f''(\xi) * (x - a)(x - b)/2$ with $\xi \in (a, b)$. Assume that $|f''| \leq M$ is bounded. Then show that $\mid \int_a^b E(x)dx \mid \leq= M/12(b - a)^3$.
6. (5+10) Consider the random vector (Y_1, Y_2, Y_3) with expectation $(\theta_1, \theta_1 + \theta_2, \theta_2)$, var($Y_i$)=1, $i = 1, 2, 3$, cov(Y_i, Y_j)=-1/2, for $i \neq j$. The objective is to estimate θ_1. One unbiased estimator is Y_1. Use the idea of control variates to propose another estimator and compare the number of simulations required for the same degree of accuracy. [It is assumed that one simulation is generation of one observation from the joint distribution of (Y_1, Y_2, Y_3) and takes the same computational effort at generating one observation from the marginal distribution of Y_1.].