INDIAN STATISTICAL INSTITUTE

M.STAT Second Year

2016-17 Semester II

Computational Finance

Final Examination

Points for each question is in brackets. Total Points 100.
Students are allowed to bring 4 pages (one-sided) of hand-written notes
Duration: 3 hours

1. (15) We have three interpolation points $(x_i, f(x_i)), i = 0, 1, 2$ with $x_i = x + (i-1)h$ and want to approximate $f'(x)$ using Lagrange interpolation. Show that this leads to the second-order centered approximation of the first-derivative, that is,
$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{1}{6}f'''(\xi)h^2$$
where $\xi \in (\min\{x, x_0, x_1, x_2\}, \max\{x, x_0, x_1, x_2\})$.

2. (20) Suppose you want to evaluate the integral of the function $f(x) = e^x$ over $[0, 1]$ using Monte Carlo simulation from Unif(0,1). Use the control variable method with $g(x) = x$ and show that the variance is reduced by a factor of 60. Is there much additional improvement if you use a general quadratic function of x?

3. (10) Describe the Mersenne twister random number generator algorithm.

4. (2+6+6+6) Consider the case of a binomial lattice for which the probability of an up move has the same value p at all nodes.
   (a) What is the distribution of the total number of up moves N through an m-step lattice?
   (b) How can you generate stratified samples from this distribution with equiprobable strata?
   (c) Show that given N, all paths through the lattice with N up moves are equally likely.
   (d) Hence outline a procedure to generate a path conditional on N.

5. (15) Let us assume that $F_1(x)$ and $F_2(x)$ are two cdf satisfying $F_1(x) \leq F_2(x)$ for all values of x. If these two distributions are proposed as models for the returns, which of these two distributions will give the larger Value-at-risk at level 0.01? Show that from the given information it is not possible to infer which distribution leads to higher expected shortfall.

6. (10+10) The process ϵ_t is said to be a strong ARCH(1) process, if $E[\epsilon_t | F_{t-1}] = 0$, $Var(\epsilon_t | F_{t-1}) = \sigma_t^2$ and $Z_t = \epsilon_t / \sigma_t$ is i.i.d. with $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2$. Assuming ϵ_t to be a strong ARCH(1) process,
   (a) Show that ϵ_t is white noise, but not independent. In particular show that the autocorrelations of ϵ_t^2 are non-zero.
   (b) If Z_t has normal distribution, show that the distribution of ϵ_t is leptokurtic.