# INDIAN STATISTICAL INSTITUTE
## M.STAT Second Year
## 2017-18 Semester II

Computational Finance
Back Paper Examination

*Points for each question is in brackets. Total Points 100.*
*Students are allowed to bring 4 pages (one-sided) of hand-written notes*
Duration: 3 hours

1. (10) Use the differentiation by integration formula with $n = 2$ while approximating the derivative at $x_1$, to obtain the second-order centered approximation of the first-derivative.

2. (20) Suppose you want to evaluate the integral of the function $f(x) = e^x$ over $[0, 1]$ using Monte Carlo simulation from Unif(0,1). Use the control variable method with $g(x) = x$ and show that the variance is reduced by a factor of 60. Is there much additional improvement if you use a general quadratic function of $x$?

3. (15) Obtain the price of a geometric average PUT option analytically when the underlying STOCK follows geometric Brownian motion.

4. (10+5+5)

   (a) For a given level $\alpha$ compute the Value at Risk and Expected shortfall under the following distributional assumption in the loss distribution of the portfolio:

      i. Exponential with rate parameter $\lambda$.

      ii. Standard Pareto distribution with shape parameter $\xi$, location parameter 0 and scale parameter 1, that is, the pdf is

      $$f_\xi(x) = (1 + \xi x)^{-(1+1/\xi)} \quad \text{if} \quad x > 0$$

   (b) For each $\alpha \in (0, 1)$, derive an equation that the rate parameter $\lambda$ and the shape parameter $\xi$ must satisfy in order for the values of VaR($\alpha$) for the two distributions to be the same.

   (c) Assuming that the parameters $\lambda$ and $\xi$ satisfy the relationship in (b) above, compare the corresponding values of the expected short fall in the two models and comment on the differences.

5. (5+10)

   (a) Describe the general state space model.

   (b) Obtain a state-space representation of the ARMA(2,1) model for a multivariate time series. Is this representation unique?

6. (5+5+10) Consider the geometric brownian motion model for stock prices with constant drift and volatility in a fixed interval of time $[0, T]$. Suppose prices are observed at time points $0 = t_0 < t_1 \cdots < t_n = T$.

   (a) Describe the set-up of infill asymptotics.

(b) What is the MLE of the drift. Is it consistent?

(c) What is the MLE of the volatility? Show that it is consistent and derive its asymptotic distribution.