Homework 3

1. The objective is to evaluate π using Monte Carlo. We proceed as follows.
   (a) Consider independent uniform(0,1) random variables, X and Y. Show that $P(X^2 + Y^2 < 1) = \pi/4$.
   (b) Evaluate π using the above result and $n = 100$ realizations of X, Y. Call it $\hat{\pi}_1$,
   (c) Repeat the procedure 1000 times and obtain the mean and variance of $\hat{\pi}_1$.
   (d) Evaluate π with $n = 10000$ realizations of X, Y. Call it $\hat{\pi}_2$.
   (e) Repeat the procedure 1000 times and obtain the mean and variance of $\hat{\pi}_2$.
   (f) Comment on the results keeping in mind the law of large numbers and central limit theorem.

2. Draw a random sample of size 5 from $\mathcal{N}(0,1)$ distribution X_1, X_2, X_3, X_4, X_5. Calculate $Y = 2X_1/\sqrt{\sum_{i=2}^{5} X_i^2}$. Repeat this 1000 times and plot the histogram of the values of Y. What is the distribution of Y? Superimpose the density of this distribution on the histogram.

3. 10 balls labelled $1, 2, \cdots, 10$ are thrown uniformly at random into 4 labelled bins (i.e., balls are thrown one by one independently into bins chosen uniformly at random).Let A be the event that the first bin contains an even numbered ball. Let B be the event that the second bin contains an odd numbered ball. Compute P(B| A) and P(B) by simulating the experiment 1000 times.

4. Use accept-reject method to generate a random variable from the following distributions starting from uniform and using inverse cdf transformation, if necessary.
   (a) $\beta(2.5, 3.5)$
   (b) $\gamma(7.2)$